Preference Aggregation by Voting: Algorithmics and Complexity

Präferenzaggregation durch Wählen: Algorithmik und Komplexität

Folien zur Vorlesung

Wintersemester 2020/2021

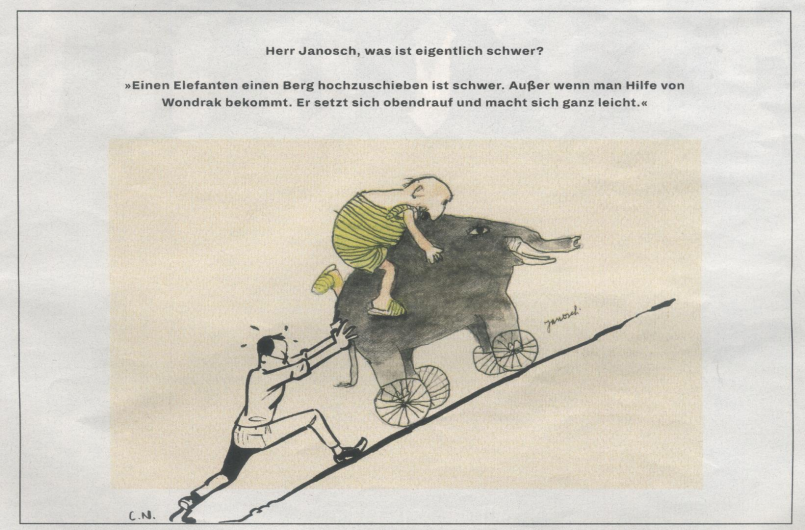
Dozent: Prof. Dr. J. Rothe

Websites

- All information for this module can be found in **ILIAS**.
- In addition, slides, exercises, and other material can be downloaded from:

  `http://ccc.cs.uni-duesseldorf.de/~rothe/wahlen`

About: Algorithmics and Complexity



About: A Really Hard & Stubborn Problem

About: Efficiency

About: Hidden Intractability

About: Algorithmics and Complexity

SIAM J. Comput.
Vol. 26, No. 3, pp. 634–653, June 1997

UNAMBIGUOUS COMPUTATION: BOOLEAN HIERARCHIES AND SPARSE TURING-COMPLETE SETS*

LANE A. HEMASPAANDRA† AND JÖRG ROTHE‡

Abstract. It is known that for any class $\mathcal{C}$ closed under *union and intersection*, the Boolean closure of $\mathcal{C}$, the Boolean hierarchy over $\mathcal{C}$, and the symmetric difference hierarchy over $\mathcal{C}$ all are equal. We prove that these equalities hold for any complexity class closed under *intersection*; in particular, they thus hold for unambiguous polynomial time (UP). In contrast to the NP case, we prove that the Hausdorff hierarchy and the nested difference hierarchy over UP both fail to capture the Boolean closure of UP in some relativized worlds.

Karp and Lipton proved that if *nondeterministic* polynomial time has sparse Turing-complete sets, then the polynomial hierarchy collapses. We establish the first consequences from the assumption that *unambiguous* polynomial time has sparse Turing-complete sets: (a) UP $\subseteq$ Low$_2$, where Low$_2$ is the second level of the low hierarchy, and (b) each level of the unambiguous polynomial hierarchy is contained one level lower in the promise unambiguous polynomial hierarchy than is otherwise known to be the case.

Key words. unambiguous computation, Boolean hierarchy, sparse Turing-complete sets

AMS subject classifications. 68Q15, 68Q10, 03D15

PII. S0097539794261970

1. Introduction. NP and NP-based hierarchies—such as the polynomial hierarchy [47, 57] and the Boolean hierarchy over NP [9, 10, 41]—have played such a

About: Complexity of Voting Problems

Exact Analysis of Dodgson Elections: Lewis Carroll's 1876 Voting System Is Complete for Parallel Access to NP

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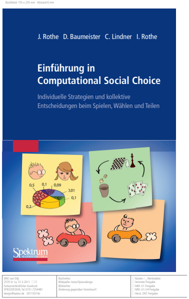
About: Preference Aggregation by Voting

About: Preference Aggregation by Voting

Literature

- **J. Rothe (Herausgeber): Economics and Computation: An Introduction to Algorithmic Game Theory, Computational Social Choice, and Fair Division.** Springer-Verlag, 2015
  with a preface by Matt O. Jackson und Yoav Shoham (Stanford)



Literature: Email from Prof. Michael Wooldridge, Oxford

Dear Joerg,

I just received a copy of "Economics and Computation". It looks FANTASTIC! I already started reading some of it, and I think we will use it on a course we are giving here next year.

It was tremendously kind of you to think about sending me a copy – I'm very grateful!

Congratulations, and thanks again!

Mike

–

Professor Michael Wooldridge mailto:mjw@cs.ox.ac.uk
Department of Computer Science, University of Oxford.
http://www.cs.ox.ac.uk/people/michael.wooldridge/

Literature: Topics

- **Foundations of Social Choice Theory**

  Election Systems and Their Properties

  Further Voting Paradoxes

  Impossibility Theorems

- **Manipulation**

  Constructive Manipulation

  Destructive Manipulation

- **Electoral Control**

  Immunity, Vulnerability, and Resistance

  Control Complexity

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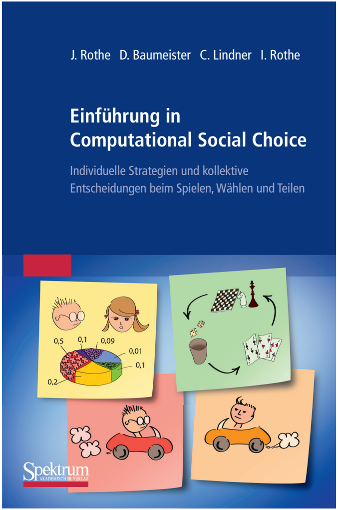

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Literature: Topics

- **Control Complexity**

  Condorcet Elections

  Approval Elections

  Bucklin and Fallback Elections

- **Single-Peaked Preferences**

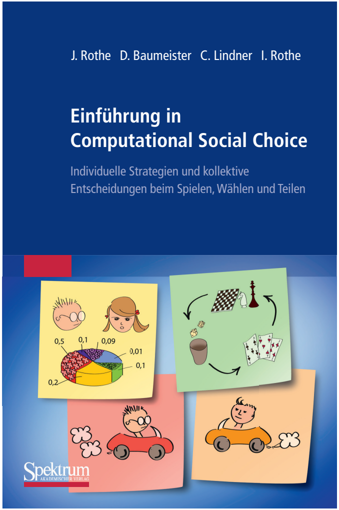
  Manipulation

  Electoral Control

- **Bribery**

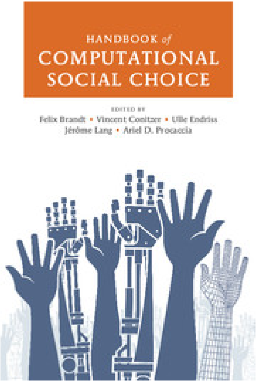
  Bribery in Copeland Elections

  Microbribery in Copeland Elections



Literature: Further Suggested Reading

- **Handbook of Computational Social Choice**, F. Brandt, V. Conitzer, U. Endriss, J. Lang und A. Procaccia (Herausgeber). Cambridge University Press, 2015



Literature: Further Suggested Reading

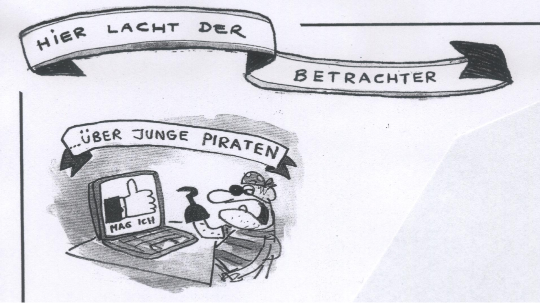
- **A Richer Understanding of the Complexity of Election Systems**, P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra und J. Rothe. Chapter 14 in *Fundamental Problems in Computing: Essays in Honor of Professor Daniel J. Rosenkrantz*, pp. 375–406, S. Ravi und S. Shukla (Herausgeber). Springer, 2009.

- **Computational Aspects of Approval Voting**, D. Baumeister, G. Erdélyi, E. Hemaspaandra, L. Hemaspaandra und J. Rothe. Chapter 10 in *Handbook on Approval Voting*, pp. 199–251, R. Sanver und J. Laslier (Herausgeber). Springer-Verlag, 2010.

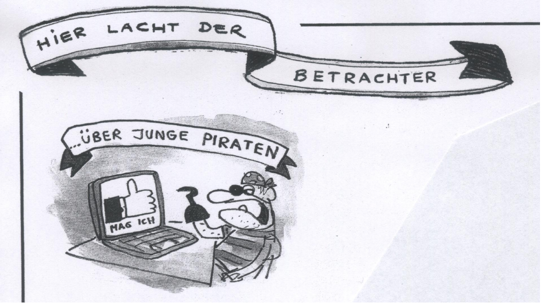
Literature: Further Suggested Reading

- **Voting Procedures**, S. Brams und P. Fishburn. Chapter 4 in Volume 1 of the *Handbook of Social Choice and Welfare*, pp. 173–236, K. Arrow, A. Sen und K. Suzumura (Herausgeber). North-Holland, 2002.
- **Chaotic Elections! A Mathematician Looks at Voting**, D. Saari. American Mathematical Society, 2001.
- **Original Papers** cited in these books and book chapters.
- ...

Computational Social Choice? Voting? Pirates?



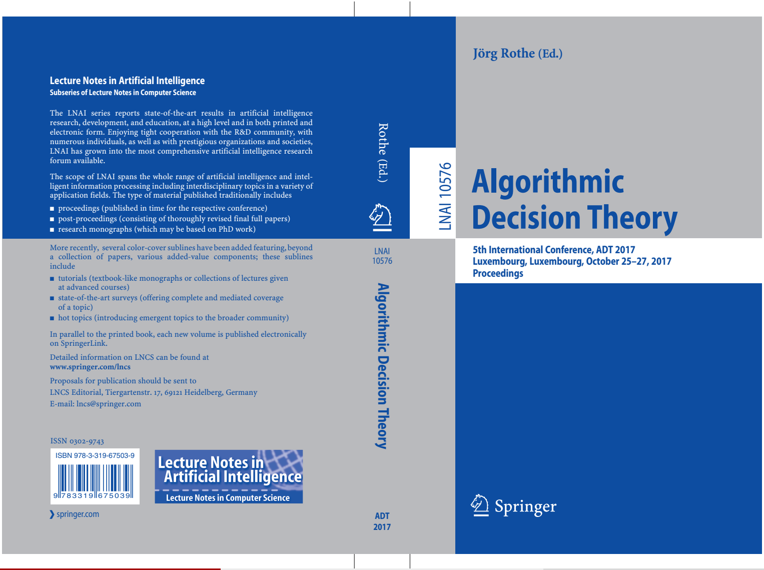
Computational Social Choice? Voting? Pirates?



Voting and Computer Science?

- At AAMAS-2017 (AAMAS is the most important multiagent systems conference), more sessions were held on computational social choice than on any other topic.
- The extent to which the growth of computational social choice has been supported by computational complexity is vividly clear when one notices that of the 21 papers in those session, fully one third had the word "complexity" in their titles.
- At AAMAS-2003, the string "social choice" does not even appear in the ACM Digital Library online table of contents; neither does the string "election" or any form of "vote," and only three papers in the entire conference have the word "complexity" in their titles.

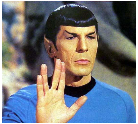
Voting and Computer Science?



Elections

The Captain of Starship Enterprise is to be elected: 

Candidates:

Voters:

Elections

Definition

- An *election* (or *preference profile*) (C, V) is specified by a set

$$C = \{c_1, c_2, \ldots, c_m\}$$

  of candidates and a list

$$V = (v_1, v_2, \ldots, v_n)$$

  of votes over C.

- How the voters' preferences are represented depends on the voting system used, e.g., by
  - a *linear order (strict ranking)* or
  - an *approval vector*.

Elections

Definition

A *linear order* (or *strict ranking*) $>$ *on* C is a binary relation on C that is

- *total*: for any two distinct $c, d \in C$, either $c > d$ or $d > c$;
- *transitive*: for all $c, d, e \in C$, if $c > d$ and $d > e$ then $c > e$;
- *asymmetric*: for all $c, d \in C$, if $c > d$ then $d > c$ does not hold.

Remark:

1. Asymmetry of $>$ implies irreflexivity of $>$.
2. We often omit the symbol $>$ in the linear orders and write, e.g.,

$$b \quad c \quad a \quad e \quad d \qquad \text{instead of} \qquad b \;>\; c \;>\; a \;>\; e \;>\; d$$

to indicate that this voter (strictly) prefers b to c, c to a, a to e, and e to d. So the leftmost candidate is the most preferred one.

Elections

Remark:

3. Occasionally, by dropping asymmetry voters are allowed to be *indifferent* between candidates, as in:

$$b > c = a > e = d$$

If so, it will be mentioned explicitly.

4. One may distinguish between *weighted* and *unweighted* voters.
Default case: unweighted voters (i.e., each voter has weight one).

5. Votes may be represented either *succinctly* or *nonsuccinctly*.
Default case: nonsuccinct (i.e., one ballot per voter).

Elections

Example

Election (C, V) with $C = \{a, b, c, d, e\}$ and $V = (v_1, \ldots, v_7)$:

$$
\begin{array}{lccccc}
v_1 : & c & b & a & e & d \\
v_2 : & a & e & d & c & b \\
v_3 : & b & a & c & e & d \\
v_4 : & b & d & e & a & c \\
v_5 : & c & b & a & e & d \\
v_6 : & c & d & b & e & a \\
v_7 : & e & d & a & b & c
\end{array}
$$

Who should win this election?

Election Systems

Definition

An *election system* (or *voting system*) is a rule determining the winner(s) of a given election (C, V). That is, it can be described by a mapping

$$f : \{(C, V) \mid (C, V) \text{ is a preference profile}\} \to 2^C,$$

a so-called *social choice correspondence*, where 2^C denotes the *power set of* C, i.e., the set of all subsets of C.

For a preference profile $P = (C, V)$, $f(P) \subseteq C$ is the *set of winners of* P, and it is possible that $f(P) = \emptyset$.

Election Systems

Remark:

- A *social choice function* is a mapping

$$f : \{(C,V) \mid (C,V) \text{ is a preference profile}\} \to C,$$

  that assigns a single winner to each given preference profile.

- A *social welfare function* describes not only how to select a winner or set of winners by a voting system, but even returns a complete ranking of the candidates. This is formalized by a mapping

$$f : \{(C,V) \mid (C,V) \text{ is a preference profile}\} \to \rho(C),$$

  where $\rho(C)$ is a ranking of (or, preference list over) the candidates in C.

Election Systems: An Incomplete Taxonomy

- **Preference-based Systems:**
  - Positional scoring protocols (plurality, veto, *k*-approval, Borda, ...)
  - Majority-based voting (simple majority, Bucklin voting, ...)
  - Pairwise-comparison-based voting procedures (Condorcet, Black, Dodgson, Young, Kemeny, Copeland, Llull, ...)
  - Point distribution voting procedures (single transferable vote, ...)
- **Nonranked Systems:**
  - Approval voting
  - Negative voting
  - Plurality voting
  - Multistage voting procedures (plurality with runoff, ...)
- **Hybrid Systems:**
  - Sincere-strategy preference-based approval voting
  - Fallback voting

Election Systems: Plurality, Antiplurality, k-Approval

Definition

- *Plurality-rule elections*: The winners are precisely those candidates who are ranked first by the most voters.
- *Antiplurality-rule (a.k.a. veto) elections*: The winners are precisely those candidates who are ranked last by the fewest voters.
- *k-approval*: Each voter gives one point to each of the k most preferred candidates. Whoever scores the most points wins.

In our above example, c is the plurality winner, e is the antiplurality winner, and both a and b are 3-approval winners.

Election Systems: Plurality, Antiplurality, k-Approval

$$
\begin{array}{lccccc}
v_1: & c & b & a & e & d \\
v_2: & a & e & d & c & b \\
v_3: & b & a & c & e & d \\
v_4: & b & d & e & a & c \\
v_5: & c & b & a & e & d \\
v_6: & c & d & b & e & a \\
v_7: & e & d & a & b & c
\end{array}
$$

- c is the plurality winner, as c has the most (namely, 3) top positions.
- e is the antiplurality (i.e., veto) winner, as e is never ranked last.
- Both a and b are 3-approval winners, as they are ranked most often (5 times) among the first three positions.

Election Systems: Borda Count

Definition

- *Borda Count*: With m candidates, each voter gives:
  - $m-1$ points to the candidate ranked at first position,
  - $m-2$ points to the candidate ranked at second position,

    ⋮
  - 0 points to the candidate ranked at last position.

  Whoever scores the most points wins.

In our above example, b is the Borda winner.

Election Systems: Borda Count

points :	4	3	2	1	0
v_1 :	c	b	a	e	d
v_2 :	a	e	d	c	b
v_3 :	b	a	c	e	d
v_4 :	b	d	e	a	c
v_5 :	c	b	a	e	d
v_6 :	c	d	b	e	a
v_7 :	e	d	a	b	c

Viewed as a social welfare function, the Borda system yields:

ranking	b	$>$	c	$>$	a	$>$	e	$>$	d
points	17	$>$	15	$>$	14	$>$	13	$>$	11

Election Systems: Scoring Protocols

Definition

A *scoring protocol* for m candidates is specified by a *scoring vector*, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)$, satisfying

$$\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m.$$

Votes are linear orders. Each vote contributes

- α_1 points to that vote's most preferred candidate,
- α_2 points to that vote's second most preferred candidate,

  $\vdots$
- α_m points to that vote's least preferred candidate.

Whoever scores the most points wins.

Election Systems: Scoring Protocols for m Candidates

Voting System	Scoring Vector
Plurality	$\alpha = (1, \overbrace{0, \ldots, 0}^{m-1})$
Antiplurality (Veto)	$\alpha = (\overbrace{1, \ldots, 1}^{m-1}, 0)$
k-Approval ($(m-k)$-Veto)	$\alpha = (\overbrace{1, \ldots, 1}^{k}, \overbrace{0, \ldots, 0}^{m-k})$
Borda Count	$\alpha = (m-1, m-2, \ldots, 0)$
$\vdots$	$\vdots$

Simple Majority and Condorcet Voting

Definition

A candidate c wins by *(simple) majority* if c is ranked first by more than half of the voters.

In our above example, no candidate wins by simple majority. This obstacle is avoided by, e.g., Bucklin voting.

Definition

A candidate c is a *Condorcet winner* if c defeats every other candidate by a strict majority in pairwise comparisons.

Simple Majority and Condorcet Voting

In our above example, there is no Condorcet winner:

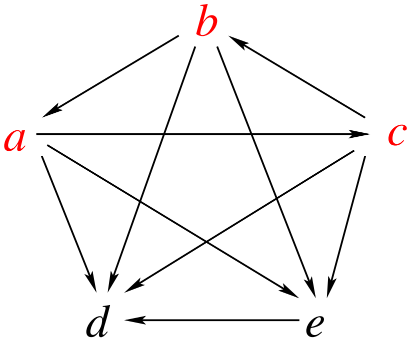
v_1 :	c	b	a	e	d
v_2 :	a	e	d	c	b
v_3 :	b	a	c	e	d
v_4 :	b	d	e	a	c
v_5 :	c	b	a	e	d
v_6 :	c	d	b	e	a
v_7 :	e	d	a	b	c

	a	b	c	d	e
a	$\times$	$2:5$	$4:3$	$4:3$	$4:3$
b		$\times$	$3:4$	$4:3$	$5:2$
c			$\times$	$4:3$	$4:3$
d				$\times$	$2:5$
e					$\times$

That is, we have a top-3-cycle among a, b, and c.

Simple Majority and Condorcet Voting

In our above example, there is no Condorcet winner:



	a	b	c	d	e
a	$\times$	2 : 5	4 : 3	4 : 3	4 : 3
b		$\times$	3 : 4	4 : 3	5 : 2
c			$\times$	4 : 3	4 : 3
d				$\times$	2 : 5
e					$\times$

That is, we have a top-3-cycle among a, b, and c. This obstacle is avoided by, e.g., Black, Dodgson, Young, Copeland, and Kemeny voting.

The Condorcet Paradox

The *Condorcet paradox* occurs whenever there exists no Condorcet winner.

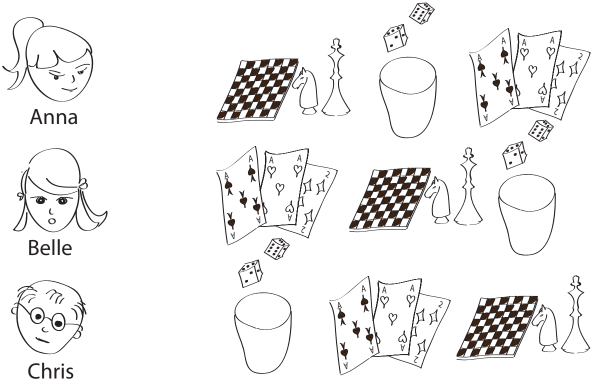


Figure: Anna, Belle, and Chris are voting on which game to play

The Condorcet Paradox

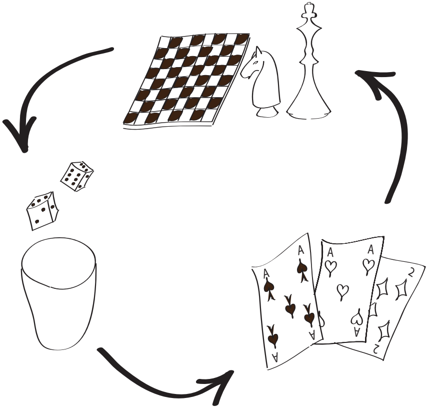


Figure: The Condorcet paradox

Majority Criterion & Condorcet Criterion

Definition

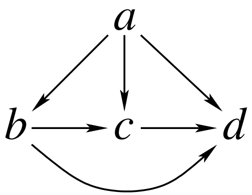
A voting system satisfies the

1. *majority criterion* if it selects the majority winner whenever one exists;
2. *Condorcet criterion* if it selects the Condorcet winner whenever one exists.

The Condorcet criterion is violated by many voting systems, e.g., by

plurality:

b	a	c	d
c	a	b	d
d	a	b	c



$\Longrightarrow$ a is the Condorcet winner but does not win under plurality.

Majority Criterion & Condorcet Criterion

A simple majority winner always also wins under plurality, so plurality satisfies the majority criterion.

However, Borda does not satisfy this criterion:

Example

4	3	2	1	0
a	b	c	d	e
a	b	c	d	e
c	b	d	e	a

a is the majority winner,

but under Borda:

a scores $2 \cdot 4 = 8$ points,

b scores $3 \cdot 3 = 9$ points and wins,

c scores $4 + 2 \cdot 2 = 8$ points,

d scores $2 + 2 \cdot 1 = 4$ points, and

e scores 1 point.

The Borda Paradox

The *Borda paradox* occurs whenever a plurality winner is the "Condorcet loser," i.e., is defeated by every other candidate in pairwise contests by a majority of votes.

Example

	2	1	0
$4\times$	a	b	c
$3\times$	b	c	a
$2\times$	c	b	a

a is the plurality winner but is defeated by b and c with $4:5$ in pairwise comparison, so a is the Condorcet loser.

In Borda:
b scores 12 points and wins,
a scores 8 points, and
c scores 7 points.

Borda's original example has 21 voters and 3 candidates.

Black Voting

Definition

Black voting:

1. Choose the Condorcet winner if there exists one.
2. Otherwise, choose all Borda winners.

Black's system:

- satisfies the Condorcet criterion and
- monotonicity, but
- it is inconsistent.

Monotonicity and Consistency

Definition

1. A voting system is *monotonic* if the following holds: If
   - some candidate w wins an election and
   - we then improve the position of w in some of the votes, leaving everything else the same,

   then w still wins in the changed election.

   The *winner-turns-loser paradox* shows failure of monotonicity.

2. A voting system is *consistent* if the following holds: When the electorate is divided arbitrarily into two (or more) parts and separate elections in each part result in the same winners, they also win an election of the entire electorate.

   The *multiple-districts paradox* shows inconsistency.

Black Is Inconsistent

Example

		2	1	0
$V_1 =$	$4\times$	a	b	c
	$3\times$	b	c	a
$V_2 =$	$3\times$	a	b	c
	$2\times$	b	c	a
	$2\times$	c	a	b

Let $C = \{a, b, c\}$.

- In (C, V_1), a is the Condorcet—and thus the Black—winner because a defeats both b and c with $4 : 3$.
- In (C, V_2), there is no Condorcet winner:
  a defeats b with $5 : 2$;
  b defeats c with $5 : 2$;
  c defeats a with $4 : 3$.
  Under Borda,
  a has a score of $\mathbf{3 \cdot 2 + 2 \cdot 1 = 8}$ and wins;
  b has a score of $2 \cdot 2 + 3 \cdot 1 = 7$;
  c has a score of $2 \cdot 2 + 2 \cdot 1 = 6$.

Black Is Inconsistent

Example

	2	1	0
$V_1 =$ $4\times$	a	b	c
$3\times$	b	c	a
$V_2 =$ $3\times$	a	b	c
$2\times$	b	c	a
$2\times$	c	a	b

So a is the Black winner in (C, V_1) and (C, V_2).

However, in $(C, V_1 \cup V_2)$,

- there is no Condorcet winner because the contest of a versus c ends in a tie: $7 : 7$; and
- under Borda,
  a has a score of $7 \cdot 2 + 2 \cdot 1 = 16$;
  b has a score of $\mathbf{5} \cdot \mathbf{2} + \mathbf{7} \cdot \mathbf{1} = \mathbf{17}$ and wins;
  c has a score of $2 \cdot 2 + 5 \cdot 1 = 9$.

Hence, b is the Black winner of $(C, V_1 \cup V_2)$.

Therefore, Black is inconsistent.

Monotonicity

Recall the definition of monotonicity:

A voting system is *monotonic* if the following holds: If

- some candidate w wins an election and
- we then improve the position of w in some of the votes, leaving everything else the same,

then w still wins in the changed election.

The *winner-turns-loser paradox* shows failure of monotonicity.

For example, changing

$$a\ b\ c\ d \quad \text{into} \quad c\ b\ a\ d$$

improves the position of c, *but it does not leave everything else the same* because it also swaps a and b.

Examples of (Non-)Monotonic Voting Systems

1. Examples of monotonic voting systems are:
   - plurality, Borda, and (more generally) all scoring protocols,
   - Condorcet,
   - Black, ...
2. Examples of nonmonotonic voting systems are:
   - **Plurality with Runoff**, using a tie-breaking rule if needed:
     - Top two candidates wrt. plurality score proceed to runoff (unless one already has an absolute majority and wins immediately);
     - the winner is whoever is ranked higher by more voters than the other.
   - **Single Transferable Vote (STV)**, which proceeds in $m-1$ rounds:
     - In each round, a candidate with lowest plurality score is eliminated (using some tie-breaking rule if needed) and all votes for this candidate transfer to the next remaining candidate in this vote's order.
     - The last remaining candidate wins.
   - **Dodgson**, (some slides ahead).

Examples of Monotonic Voting Systems

1. **Plurality is monotonic:**
   - Improving the position of c can only increase c's plurality score.
   - Since everything else stays the same, the plurality score of all other candidates can only get worse.

   The same argument works to show that Borda and (more generally) all scoring protocols are monotonic because of

$$\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m.$$

2. **Condorcet is monotonic:** If there exists a Condorcet winner c, c remains the Condorcet winner in the election where c's position is improved and everything else is left the same.

3. **Black is monotonic:** follows immediately from the monotonicity of Condorcet and Borda.

A Stronger Notion of Monotonicity

Definition

A voting system is *strongly monotonic* if the following holds: If

- some candidate w wins an election and
- we then change the votes in such a way that every candidate originally ranked behind w is still ranked behind w after the change,

then w still wins in the changed election.

For example, **plurality is not strongly monotonic:**

$3\times$	a	b	c		a	b	c
$2\times$	b	c	a	$\Longrightarrow$	b	c	a
$2\times$	c	b	a		b	c	a
		a wins				b wins	

Plurality with Runoff Is Not Monotonic

Example

$27\times$	a	b	c	Runoff: a and c, c wins $66:27$
$42\times$	c	a	b	
$24\times$	b	c	a	

Change the election as follows: 4 of the 27 voters improve c's position, and we obtain:

$23\times$	a	b	c	Runoff: b and c, b wins $47:46$
$46\times$	c	a	b	
$24\times$	b	c	a	

Hence, plurality with runoff is not monotonic.

Remark: Plurality with runoff with three candidates is the same as STV; still we give another example for STV on the next slide.

STV Is Not Monotonic

Example

$7\times$	a	b	c	Eliminate c; transfer c's votes to a; a wins $13:7$ against b
$6\times$	c	a	b	
$7\times$	b	c	a	

Change the election: 2 of the $b\ c\ a$ votes improve a to $a\ b\ c$, and we obtain:

$9\times$	a	b	c	Eliminate b; transfer b's votes to c; c wins $11:9$ against a
$6\times$	c	a	b	
$5\times$	b	c	a	

Thus single transferable vote is not monotonic.

Remark: Again, this also works as a counterexample for plurality with runoff, which for three candidates is the same as STV.

Condorcet Systems: Dodgson, Young, and Copeland

Let (C, V) be a given election where votes are linear orders.

- **Dodgson**: The *Dodgson score of $c \in C$* (denoted by $DScore(c)$) is the smallest number of sequential swaps needed to make c a Condorcet winner. Whoever has the smallest Dodgson score wins.
- **Young**: The *Young score of $c \in C$* (denoted by $YScore(c)$) is the size of a largest sublist of V for which c is a Condorcet winner. Whoever has the maximum Young score wins.
- **Copeland**: For each $c, d \in C$, $c \neq d$, let $N(c, d)$ be the number of voters who prefer c to d. Let $Z(c, d) = 1$ if $N(c, d) > N(d, c)$ and $Z(c, d) = 1/2$ if $N(c, d) = N(d, c)$.
  The *Copeland score of c* is $CScore(c) = \sum_{d \neq c} Z(c, d)$.
  Whoever has the maximum Copeland score wins.

Dodgson Voting Fails Monotonicity

Example (Fishburn (1977))

	Original Votes					Changed Votes			
15 votes :	c	a	d	b		c	a	d	b
9 votes :	b	d	c	a		b	d	c	a
9 votes :	a	b	d	c	$\Rightarrow$	a	b	d	c
5 votes :	a	c	b	d		a	c	b	d
5 votes :	b	a	c	d		a	b	c	d
	Dodgson winner a (3 swaps)					Dodgson winner c (2 swaps)			

Dodgson Voting Fails Monotonicity

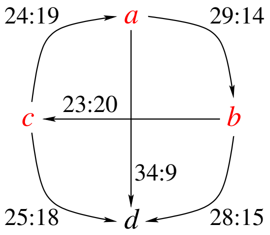
Example (Fishburn (1977))

	Original Votes			
15 votes :	c	a	d	b
9 votes :	b	d	c	a
9 votes :	a	b	d	c
5 votes :	a	c	b	d
5 votes :	b	a	c	d

Dodgson

winner a

(3 swaps)



$\Longrightarrow$ No Condorcet winner. But a becomes the Condorcet winner with three swaps: $3\times$: $c\ a\ d\ b \rightsquigarrow a\ c\ d\ b$.

$\Longrightarrow$ a defeats c with $22 : 21$. Because of deficit 5, no two swaps are enough.

$\Longrightarrow$ $DScore(a) = 3$.

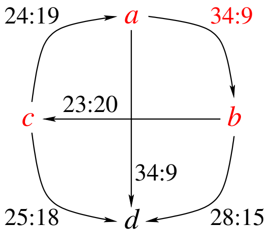
Exercise: $DScore$ of b, c, d is > 3.

Dodgson Voting Fails Monotonicity

Example (Fishburn (1977))

	Changed Votes			
15 votes :	c	a	d	b
9 votes :	b	d	c	a
9 votes :	a	b	d	c
5 votes :	a	c	b	d
5 votes :	a	b	c	d

Dodgson winner c (2 swaps)



$\Longrightarrow$ No Condorcet winner. But now c becomes the Condorcet winner with two swaps: $2\times : a\ b\ c\ d \ \rightsquigarrow\ a\ c\ b\ d$.

$\Longrightarrow$ c defeats b with $22 : 21$. Because of deficit 3, no single swap is enough.

$\Longrightarrow$ $DScore(c) = 2$.

Exercise: $DScore$ of a, b, d is > 2.

Determining Young Winners

Example

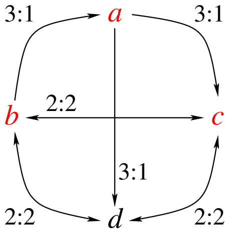
Consider the election (C, V) with $C = \{a, b, c, d\}$ and V:

v_1 : c b a d

v_2 : a d c b

v_3 : b a c d

v_4 : d b a c



- No Condorcet winner.
- a is Condorcet winner for (v_2), but in (v_2, v_3) there is a tie with b; with v_1 or v_4 even worse.
- b is Condorcet winner for (v_1, v_3, v_4), so $YScore(b) = 3$.
- c and d: even worse than a in pairwise comparison.
- Thus b is the Young winner.

Determining Copeland Winners

Example

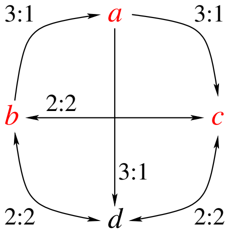
Consider the election (C, V) with $C = \{a, b, c, d\}$ and V:

v_1 : c b a d

v_2 : a d c b

v_3 : b a c d

v_4 : d b a c



- No Condorcet winner.

	a	b	c	d	$CScore$
a	$\times$	0	1	1	**2**
b	1	$\times$	$1/2$	$1/2$	**2**
c	0	$1/2$	$\times$	$1/2$	1
d	0	$1/2$	$1/2$	$\times$	1

$\Longrightarrow$ a and b are the Copeland winners.

How Hard is it to Determine Copeland, Dodgson, and Young Winners?

Fact

Copeland winners can be determined in polynomial time.

Theorem (Hemaspaandra, Hemaspaandra, and Rothe (1997))

The problem of determining Dodgson winners is complete for "parallel access to NP." **without proof**

Theorem (Rothe, Spakowski, and Vogel (2003))

The problem of determining Young winners is complete for "parallel access to NP." **without proof**

Complexity of Determining Dodgson Winners

Exact Analysis of Dodgson Elections: Lewis Carroll's 1876 Voting System Is Complete for Parallel Access to NP

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AND

JÖRG ROTHE

Friedrich-Schiller-Universität Jena, Jena, Germany

An Incomplete Summary

	Majority	Condorcet	Consistent	Monotonic
a) Plurality	✓	✗		✓
b) Borda	✗			✓
c) Veto				✓
d) Condorcet		✓		✓
e) Copeland		✓		
f) Dodgson		✓		✗
g) Young		✓		
h) Black		✓	✗	✓
i) Plurality w. Runoff				✗
j) STV				✗

Homogeneity

Definition

A voting system f is said to be *homogeneous* if for each preference profile (C, V) and for all positive integers q, it holds that

$$f((C, V)) = f((C, qV)),$$

where qV denotes V replicated q times.

Remark:

- Dodgson's system is not homogeneous.
- Fishburn (1977) proposed the following limit device to define a homogeneous variant of Dodgson elections:

$$DScore^{*}_{(C,V)}(c) = \lim_{q \to \infty} \frac{DScore_{(C,qV)}(c)}{q}.$$

Dodgson Fails Homogeneity

Example (Fishburn (1977))

Original Profile						Changed Profile				
2 votes :	d	c	a	b		6 votes :	d	c	a	b
2 votes :	b	c	a	d		6 votes :	b	c	a	d
2 votes :	c	a	b	d		6 votes :	c	a	b	d
2 votes :	d	b	c	a	$\Rightarrow$	6 votes :	d	b	c	a
2 votes :	a	b	c	d		6 votes :	a	b	c	d
1 vote :	a	d	b	c		3 votes :	a	d	b	c
1 vote :	d	a	b	c		3 votes :	d	a	b	c
Dodgson winner a						Dodgson winner $\boldsymbol{d}$				
(3 swaps)						(6 swaps)				

Dodgson Fails Homogeneity

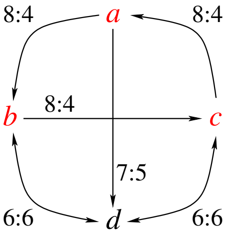
Example (Fishburn (1977))

Original Profile

2 votes :	d	c	a	b
2 votes :	b	c	a	d
2 votes :	c	a	b	d
2 votes :	d	b	c	a
2 votes :	a	b	c	d
1 vote :	a	d	b	c
1 vote :	d	a	b	c

Dodgson winner a

(3 swaps)



- No Condorcet winner.
- a becomes a Condorcet winner with 3 swaps in the first 3 votes and wins $7:5$ against c.
- No two swaps are enough.
- Thus $DScore(a) = 3$.

Dodgson Fails Homogeneity

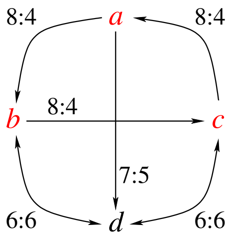
Example (Fishburn (1977))

Original Profile

2 votes :	d	c	a	b
2 votes :	b	c	a	d
2 votes :	c	a	b	d
2 votes :	d	b	c	a
2 votes :	a	b	c	d
1 vote :	a	d	b	c
1 vote :	d	a	b	c

Dodgson winner a

(3 swaps)



- $DScore(x) > 3$ for $x \in \{b, c, d\}$:
  - b needs 3 swaps against a and 1 against d;
  - c needs 3 swaps against b and 1 against d;
  - d needs 2 swaps against a, 1 against b, and 1 against c.

Dodgson Fails Homogeneity

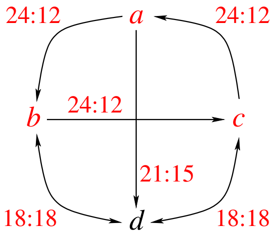
Example (Fishburn (1977))

Changed Profile

6 votes :	d	c	a	b
6 votes :	b	c	a	d
6 votes :	c	a	b	d
6 votes :	d	b	c	a
6 votes :	a	b	c	d
3 votes :	a	d	b	c
3 votes :	d	a	b	c

Dodgson winner ***d***

(6 swaps)



- 6 swaps make d a Condorcet winner:

$4\times$	b	c	a	d
$4\times$	b	c	d	a
$1\times$	b	d	c	a
$1\times$	d	b	c	a

- So d wins $19 : 17$ against each of a, b, c.

Dodgson Fails Homogeneity

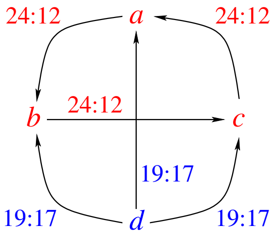
Example (Fishburn (1977))

Changed Profile

6 votes :	d	c	a	b
6 votes :	b	c	a	d
6 votes :	c	a	b	d
6 votes :	d	b	c	a
6 votes :	a	b	c	d
3 votes :	a	d	b	c
3 votes :	d	a	b	c

Dodgson winner ***d***

(6 swaps)



- 6 swaps make d a Condorcet winner:

$4\times$	b	c	a	d	
$4\times$	b	c	d	a	↲
$1\times$	b	d	c	a	↲
$1\times$	d	b	c	a	↲

- So d wins 19 : 17 against each of a, b, c.
- No 5 swaps are enough.
- Thus $DScore(d) = 6$.

Dodgson Fails Homogeneity

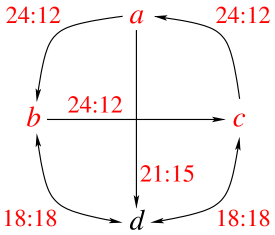
Example (Fishburn (1977))

Changed Profile

6 votes :	d	c	a	b
6 votes :	b	c	a	d
6 votes :	c	a	b	d
6 votes :	d	b	c	a
6 votes :	a	b	c	d
3 votes :	a	d	b	c
3 votes :	d	a	b	c

Dodgson winner ***d***

(6 swaps)



- $DScore(x) > 6$ for $x \in \{a, b, c\}$:
  - a needs 7 swaps against c;
  - b needs 7 swaps against a and 1 against d;
  - c needs 7 swaps against b and 1 against d;
- Thus d is the Dodgson winner.

Weak Condorcet and Weak Dodgson

Definition

- Let (C, V) be an election.

  A candidate $c \in C$ is a *weak Condorcet winner* if c ties or defeats every other candidate in pairwise comparison.

- **Weak Dodgson**:
  - The *weak Dodgson score of $c \in C$* (denoted by $\overline{DScore}_{(C,V)}(c)$) is the smallest number of sequential swaps needed to make c a weak Condorcet winner. (Also, let $\overline{DScore}^{*}_{(C,V)}(c) = \lim_{q \to \infty} \frac{\overline{DScore}_{(C,qV)}(c)}{q}$.)
  - Whoever has the smallest weak Dodgson score wins.

Remark: For an odd number of voters, the notions of Condercet winner and weak Condercet winner and, consequently, the notions of Dodgson winner and weak Dodgson winner are identical.

Weak Dodgson Fails Homogeneity

Example (Fishburn (1977))

Consider the election (C, V) with $C = \{a_1, a_2, \ldots, a_7, c\}$ and V:

a_1	a_2	a_3	a_4	c	a_5	a_6	a_7
a_7	a_1	a_2	a_3	c	a_4	a_5	a_6
a_6	a_7	a_1	a_2	c	a_3	a_4	a_5
a_5	a_6	a_7	a_1	c	a_2	a_3	a_4
a_4	a_5	a_6	a_7	c	a_1	a_2	a_3
a_3	a_4	a_5	a_6	c	a_7	a_1	a_2
a_2	a_3	a_4	a_5	c	a_6	a_7	a_1

- $\overline{DScore}_{(C,V)}(c) = 7$ and $\overline{DScore}_{(C,V)}(a_i) = 6$ for $1 \leq i \leq 7$.
- $\overline{DScore}^{*}_{(C,V)}(c) = 3.5$ and $\overline{DScore}^{*}_{(C,V)}(a_i) = 4.5$ for $1 \leq i \leq 7$, which implies that c wins in (C, qV) for large enough q.

Weak Dodgson Fails Homogeneity

a_1	a_2	a_3	a_4	c	a_5	a_6	a_7
a_7	a_1	a_2	a_3	c	a_4	a_5	a_6
a_6	a_7	a_1	a_2	c	a_3	a_4	a_5
a_5	a_6	a_7	a_1	c	a_2	a_3	a_4
a_4	a_5	a_6	a_7	c	a_1	a_2	a_3
a_3	a_4	a_5	a_6	c	a_7	a_1	a_2
a_2	a_3	a_4	a_5	c	a_6	a_7	a_1

- Swap c once to the left in each voter.
- This makes c a (weak) Condorcet winner.
- Since these 7 swaps are necessary for that, $\overline{DScore}_{(C,V)}(c) = 7$.

- a_1 defeats a_2, a_3, a_4, c with $6:1$, $5:2$, $4:3$, $4:3$, and is defeated by a_5, a_6, a_7 with $3:4$, $2:5$, $1:6$.
- 6 swaps make a_1 a (weak) Condorcet winner (and 5 are not enough):
  - $a_5\ a_6\ a_7\ a_1\ c\ a_2\ a_3\ a_4 \overset{\text{3 swaps}}{\rightsquigarrow} a_1\ a_5\ a_6\ a_7\ c\ a_2\ a_3\ a_4$
  - $a_3\ a_4\ a_5\ a_6\ c\ a_7\ a_1\ a_2 \overset{\text{1 swap}}{\rightsquigarrow} a_3\ a_4\ a_5\ a_6\ c\ a_1\ a_7\ a_2$
  - $a_2\ a_3\ a_4\ a_5\ c\ a_6\ a_7\ a_1 \overset{\text{2 swaps}}{\rightsquigarrow} a_2\ a_3\ a_4\ a_5\ c\ a_1\ a_6\ a_7$
- Thus $\overline{DScore}_{(C,V)}(a_1) = 6$, and analogously so for $a_2, a_3, a_4, a_5, a_6, a_7$.

Weak Dodgson Fails Homogeneity

a_1	a_2	a_3	a_4	c	a_5	a_6	a_7
a_7	a_1	a_2	a_3	c	a_4	a_5	a_6
a_6	a_7	a_1	a_2	c	a_3	a_4	a_5
a_5	a_6	a_7	a_1	c	a_2	a_3	a_4
a_4	a_5	a_6	a_7	c	a_1	a_2	a_3
a_3	a_4	a_5	a_6	c	a_7	a_1	a_2
a_2	a_3	a_4	a_5	c	a_6	a_7	a_1

- Swap c once to the left in each voter.
- This makes c a (weak) Condorcet winner.
- Since these 7 swaps are necessary for that, $\overline{DScore}_{(C,V)}(c) = 7$.

- Thus $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are the (weak) Dodgson winners.
- Now consider $\overline{DScore}^{*}_{(C,V)}(c)$ and $\overline{DScore}^{*}_{(C,V)}(a_i)$:

q	1	2	3	4	5	6	...
$c : a_i$	$3:4$	$6:8$	$9:12$	$12:16$	$15:20$	$18:24$	$\cdots$
$\frac{\overline{DScore}_{(C,qV)}(c)}{q}$	$\frac{7}{1}$	$\frac{7}{2} = 3.5$	$\frac{14}{3} \approx 4.\bar{6}$	$\frac{14}{4} = 3.5$	$\frac{21}{5} = 4.2$	$\frac{21}{6} = 3.5$	$\cdots$

Weak Dodgson Fails Homogeneity

a_1	a_2	a_3	a_4	c	a_5	a_6	a_7
a_7	a_1	a_2	a_3	c	a_4	a_5	a_6
a_6	a_7	a_1	a_2	c	a_3	a_4	a_5
a_5	a_6	a_7	a_1	c	a_2	a_3	a_4
a_4	a_5	a_6	a_7	c	a_1	a_2	a_3
a_3	a_4	a_5	a_6	c	a_7	a_1	a_2
a_2	a_3	a_4	a_5	c	a_6	a_7	a_1

- Then $\overline{DScore}^*_{(C,V)}(c) = 3.5$.
- Analogously, $\overline{DScore}^*_{(C,V)}(a_i) = 4.5$ for $1 \leq i \leq 7$: On average (per multiplication), a_1 needs 2.5 swaps against a_7, 1.5 swaps against a_6, and 0.5 swaps against a_5, etc.

- Thus $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are the (weak) Dodgson winners.
- Now consider $\overline{DScore}^*_{(C,V)}(c)$ and $\overline{DScore}^*_{(C,V)}(a_i)$:

q	1	2	3	4	5	6	...
$c : a_i$	$3:4$	$6:8$	$9:12$	$12:16$	$15:20$	$18:24$	$\cdots$
$\frac{\overline{DScore}_{(C,qV)}(c)}{q}$	$\frac{7}{1}$	$\frac{7}{2} = 3.5$	$\frac{14}{3} \approx 4.\bar{6}$	$\frac{14}{4} = 3.5$	$\frac{21}{5} = 4.2$	$\frac{21}{6} = 3.5$	$\cdots$

Independence of Clones

Independence of Clones

Definition

- Two candidates are *clones of each other* if they are ranked next to each other in every individual ranking, i.e., both candidates perform identically in pairwise comparisons with any other alternative.






Independence of Clones

Definition

- Two candidates are *clones of each other* if they are ranked next to each other in every individual ranking, i.e., both candidates perform identically in pairwise comparisons with any other alternative.






- A voting system is *independent of clones* if a losing candidate cannot be made a winning candidate by introducing clones.

Tideman's Example of Cloning

Example (Tideman (1987))

"When I was 12 years old I was nominated to be treasurer of my class at school. A girl named Michelle was also nominated. I relished the prospect of being treasurer, so I made a quick calculation and nominated Michelle's best friend, Charlotte. In the ensuing election

- *I received 13 votes,*
- *Michelle received 12, and*
- *Charlotte received 11,*

so I became treasurer."

In other words, Tideman cloned Michelle.

Cloning in Florida in 2000

In the 2000 US Presidential Election, Ralph Nader (Green Party) split votes away from Al Gore (Democrats), thus allowing George W. Bush (Republicans) to win the election.

The final count in Florida was:

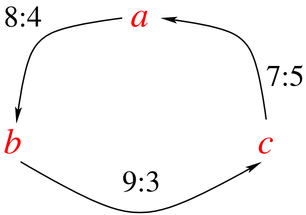
Republican	2,912,790	Workers World	1,804
Democratic	2,912,253	Constitution	1,371
Green Party	97,488	Socialist	622
Natural Law	2,281	Socialist Workers	562
Reform	17,484	Write-in	40
Libertarian	16,415		

Dodgson is Not Independent of Clones

Example (Brandt (2009))

	Original				Cloning c			
5 votes :	a	b	c		a	b	c	c'
4 votes :	b	c	a	$\Rightarrow$	b	c	c'	a
3 votes :	c	a	b		c	c'	a	b
	Dodgson winner a (2 swaps)				Dodgson winner b (3 swaps)			

Dodgson is Not Independent of Clones



	Original		
5 votes :	a	b	c
4 votes :	b	c	a
3 votes :	c	a	b
	Dodgson winner a (2 swaps)		

- No Condorcet winner.
- a is make a Condorcet winner by 2 swaps (and 1 swap is not enough):

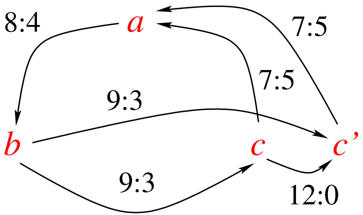
$$2\times \quad b \quad c \quad a \quad \rightsquigarrow \quad b \quad a \quad c$$

- Thus $DScore(a) = 2$. Similarly, $DScore(b) = 3$ and $DScore(c) = 4$.
- So a is the Dodgson winner.

Dodgson is Not Independent of Clones

	Cloning c			
5 votes :	a	b	c	c'
4 votes :	b	c	c'	a
3 votes :	c	c'	a	b

Dodgson winner b (3 swaps)



- But now a would need 4 swaps to defeat both c and c' and become a Condorcet winner (3 are not enough):

$$2\times \quad b \quad c \quad c' \quad a \quad \rightsquigarrow \quad b \quad a \quad c \quad c'$$

- Thus $DScore(a) = 4$, $DScore(b) = 3$, $DScore(c) = 4$, and $DScore(c') = 11$.
- So now b is the Dodgson winner.

Dodgson May Choose the Condorcet Loser and Fails the Reversal Symmetry Criterion

Definition

- Recall: A candidate c is a *Condorcet loser* if c is defeated by every other candidate by a strict majority in pairwise comparisons.
- A voting systems satisfies the *reversal symmetry criterion* if it holds that a unique winner becomes a nonwinner whenever all individual rankings are reversed.

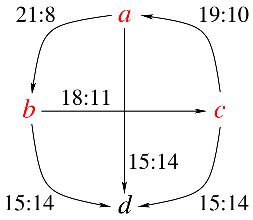
Dodgson May Choose the Condorcet Loser and Fails the Reversal Symmetry Criterion

Example (Brandt (2009))

Dodgson chooses the Condorcet loser					Dodgson fails the reversal symmetry criterion				
10 votes :	d	a	b	c	10 votes :	c	b	a	d
8 votes :	b	c	a	d	8 votes :	d	a	c	b
7 votes :	c	a	b	d	7 votes :	d	b	a	c
4 votes :	d	c	a	b	4 votes :	b	a	c	d
Dodgson winner d (3 swaps)					Dodgson winner d (no swaps)				

Dodgson May Choose the Condorcet Loser

Dodgson chooses the Condorcet loser				
10 votes :	d	a	b	c
8 votes :	b	c	a	d
7 votes :	c	a	b	d
4 votes :	d	c	a	b
Dodgson winner d (3 swaps)				



- d is the Condorcet loser and the Dodgson winner with $DScore(d) = 3$:

  $1\times$ b c a d $\rightsquigarrow$ b d c a

  $1\times$ c a b d $\rightsquigarrow$ c a d b
- $DScore(a) = 5$: $5\times$ $b\ c\ a\ d \rightsquigarrow b\ a\ c\ d$
- $DScore(b) = 7$: $7\times$ $d\ a\ b\ c \rightsquigarrow d\ b\ a\ c$
- $DScore(c) = 4$: $4\times$ $d\ a\ b\ c \rightsquigarrow d\ a\ c\ b$

Reversal Symmetry Criterion

- Plurality fails the reversal symmetry criterion:

3 votes :	a	b	c	d	d	c	b	a
2 votes :	d	c	b	a	a	b	c	d
2 votes :	b	c	d	a	a	d	c	b
	plurality winner a				plurality winner a			

- However, simple majority and Condorcet satisfy the reversal symmetry criterion.

The No Show Paradox and the Twin Paradox

Definition

- The *no show paradox* occurs whenever a voter is better off not showing up (as this leads to the election of a candidate this voter prefers). Or, more formally, adding identical preferences with c ranked last makes c win.

  A voting systems satisfies the *participation* criterion if the no show paradox never occurs.

- The *twin paradox* occurs if whenever a voter is joined by a "twin" (a voter with identical preferences), this gives less weight to their joint preferences.

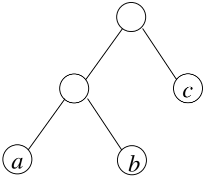
  A voting systems satisfies the *twins welcome* criterion if the twin paradox never occurs.

The No Show Paradox

Example (Moulin (1985))

Successive Elimination (Regular Cup):

Balanced binary tree whose leaves are labeled by the candidates. Each inner node is labeled by the winner of both children, where each vote is taken by majority. The candidate at the root wins.



v_1 :	c	b	a
v_2 :	c	b	a
v_3 :	a	b	c
v_4 :	a	b	c
v_5 :	c	a	b
v_6 :	b	c	a
v_7 :	b	c	a

Here: a against b, next the winner against c.

Ties are broken lexicographically: $a > b > c$.

The No Show Paradox

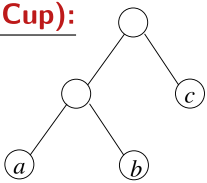
Example (Moulin (1985))

Successive Elimination (Regular Cup):

Round 1: b defeats a with $4 : 3$.

Round 2: b defeats c with $4 : 3$.

$\Longrightarrow$ b wins.



v_1 :	c	b	a
v_2 :	c	b	a
v_3 :	a	b	c
v_4 :	a	b	c
v_5 :	c	a	b
v_6 :	b	c	a
v_7 :	b	c	a

Here: a against b, next the winner against c.

Ties are broken lexicographically: $a > b > c$.

The No Show Paradox

Example (Moulin (1985))

Successive Elimination (Regular Cup):

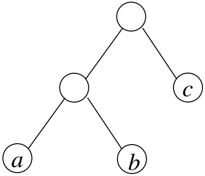
Assume v_1 doesn't show up.

Round 1: a defeats b with $3:3$ and tie-breaking.

Round 2: c defeats a with $4:2$.

$\Longrightarrow$ c wins.

Since v_1 prefers c to b, v_1 better stays home.



v_2 :	c	b	a
v_3 :	a	b	c
v_4 :	a	b	c
v_5 :	c	a	b
v_6 :	b	c	a
v_7 :	b	c	a

Here: a against b, next the winner against c.

Ties are broken lexicographically: $a > b > c$.

The Twin Paradox

Example (Moulin (1985))

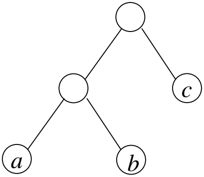
Successive Elimination (Regular Cup):

Consider again (C, V) with $V = \{v_2, \ldots v_7\}$.

As we have seen, c wins.

Is v_2 glad to see the twin v_1 participate?

NO! As we have seen, then b wins, but v_2 (like v_1) prefers c to b.



v_2 :	c	b	a
v_3 :	a	b	c
v_4 :	a	b	c
v_5 :	c	a	b
v_6 :	b	c	a
v_7 :	b	c	a

Here: a against b, next the winner against c.
Ties are broken lexicographically: $a > b > c$.

The No Show Paradox and the Twin Paradox

Remark:

- Voting systems immune to both paradoxes include:
  - plurality, Borda, and (more generally) all scoring protocols,
  - simple majority.
- Voting systems subject to the no show paradox include:
  - plurality with runoff,
  - successive elimination.

Fact

If a voting system is immune to the no show paradox, it is also immune to the twin paradox.

The No Show Paradox and the Twin Paradox

Theorem (Moulin (1988))

1. *For at most three candidates, there exist voting systems satisfying the Condorcet and participation criteria.*
2. *For at least four candidates (and at least 25 voters), no voting system satisfies the Condorcet and participation criteria.*

without proof

Further Properties of Voting Systems

Definition

A voting system is

- *anonymous* if it treats all voters equally: if any two voters trade their ballots, the outcome remains the same;
- *neutral* if it treats all candidates equally: if any two candidates are swapped in each vote, the outcome changes accordingly;
- *onto* (satisfies *citizens' sovereignty*) if for each candidate there are some votes that would make that candidate win;
- *nondictatorial* if there does not exist a dictator (i.e., a voter whose most preferred candidate always wins);
- *resolute* (*single-valued*) if it always selects a single candidate as the winner.

Further Properties of Voting Systems

Definition

- A voting system satisfies the *Pareto condition*: If c is ranked above d in all votes then the system ranks c above d;
- A voting system is *independent of irrelevant alternatives (Arrow's IIA)* if the social preferences between any two candidates c and d depend only on the individual preferences between c and d: If
  - the system ranks c above d and
  - we then change the votes but not who of c and d is ranked better,

  then the system should still rank c above d.

All our systems so far satisfy each of these conditions, except resoluteness and Arrow's IIA.

Arrow's Impossibility Theorem

Theorem (Arrow (1951))

Suppose there are at least three candidates.

There exists no voting system that simultaneously:

- *satisfies the Pareto condition,*
- *is independent of irrelevant alternatives, and*
- *nondictatorial.*

without proof

Muller–Satterthwaite Impossibility Theorem

Theorem (Muller and Satterthwaite (1977))

Suppose there are at least three candidates.

There exists no voting system that simultaneously is:

- *resolute,*
- *onto,*
- *strongly monotonic, and*
- *nondictatorial.*

without proof

Gibbard–Satterthwaite Impossibility Theorem

Theorem (Gibbard (1973) and Satterthwaite (1975))

Suppose there are at least three candidates.

There exists no voting system that simultaneously is:

- *resolute,*
- *onto,*
- *nondictatorial, and*
- *nonmanipulable.*

without proof

Remark: Intuitively, a voting system is *manipulable* if some voter can be better off revealing his or her vote insincerely.

Summary of Properties of Voting Systems

	anonymity	neutrality	nondictatorship	Pareto consistency	citizens' sovereignty	majority criterion	Condorcet criterion	monotonicity	homogeneity	IIA	independence of clones	strategy-proofness	participation criterion	consistency
plurality	✓	✓	✓	✓	✓	✓	✗	✓	✓	✗	✗	✗	✓	✓
Borda	✓	✓	✓	✓	✓	✗	✗	✓	✓	✗	✗	✗	✓	✓
Copeland	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗
Dodgson	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗	✗	✗
Young	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗	✗
STV	✓	✓	✓	✓	✓	✓	✗	✗	✓	✗	✓	✗	✗	✗