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Lecture in Summer Term 2026

Example Exam Algorithmic Game Theory

July 21st 2026

Please do not write with a pencil or a red pen!
Enter your surname, first name and matriculation number on each sheet and also your major and semester on the cover sheet!

Surname, First name:

Major, Semester:

Matriculation number:

Number of additional sheets:

Permitted resources:

- lecture notes, books, exercise sheets,
- your memory.

Not permitted resources:

- cell phones and other communication devices,
- other students,
- programmable calculators.

Make sure that your calculations and all intermediate steps are complete and clear!

Task	1	2	3	4	5	6	Bonus	Total
Possible points	20	22	10	18	20	10	5 (Bonus)	105
Your points								

Task 1 (20 points) *Multiple Choice*

For each of the following questions, check either „Yes“ or „No“ in each row.

Evaluation: Let $\#R$ denote the number of answers that you have checked correctly and let $\#K$ be the total number of answers that you have checked (counting only those with *either „Yes“ or „No“* – answers with *neither „Yes“ nor „No“* or *both „Yes“ and „No“* are not counted in $\#K$). Then you receive the following total number of points:

$$\#R + \left\lfloor \frac{5 \cdot \#R}{\#K} \right\rfloor \text{ points if } \#K > 0\text{, and } 0 \text{ points if } \#K = 0.$$

(a) Which of the following statements is/are true in noncooperative 2-player normal-form games?

- ☐ Yes ☐ No Every strict Nash equilibrium in pure strategies is a profile of strictly dominant strategies.
- ☐ Yes ☐ No Every Pareto-optimal profile contains only dominant strategies.
- ☐ Yes ☐ No A Nash equilibrium in pure strategies does not always exist.

(b) Which of the following statements is/are true?

- ☐ Yes ☐ No Every 2-simplex has three 0-*faces* and three 1-*faces*.
- ☐ Yes ☐ No The problem GEOGRAPHY (given a graph and a start vertex, is there a winning strategy for player 1 in the game *Geography*?) is PSPACE-complete.
- ☐ Yes ☐ No The 3-player majority game $G = (\{1, 2, 3\}, v)$ (defined by $v(C) = 1$ if $\|C\| \geq 2$, and $v(C) = 0$ otherwise) is neither monotonic nor anonymous.

(c) Which of the following statements is/are true?

- ☐ Yes ☐ No Define the cooperative game $G = (\{1, 2, 3, 4\}, v)$ by $v(C) = 1$ if $\|C\| \geq 2$, and $v(C) = 0$ otherwise. G is superadditive.
- ☐ Yes ☐ No In cooperative games, a veto player does not always exist.
- ☐ Yes ☐ No There are convex cooperative games with an empty core.

(d) Which of the following statements is/are true?

- ☐ Yes ☐ No A simple superadditive game has a nonempty core if and only if it has a veto player.
- ☐ Yes ☐ No The problem DUMMY (given a weighted voting game G and a player i, is i a dummy player in G?) is coNP-complete.
- ☐ Yes ☐ No If a player has Shapley value 0 in a nonmonotonic cooperative game, then he must be a dummy player.

(e) Consider the weighted voting game $G = (2, 2, 2; 5)$. Which of the following statements is/are true?

- ☐ Yes ☐ No The Shapley–Shubik index of all three players in G is $\frac{1}{3}$.
- ☐ Yes ☐ No The normalized Banzhaf index of all three players in G is $\frac{1}{2}$.
- ☐ Yes ☐ No If the players 1 and 2 are merged to one player of weight 4, then this player's probabilistic Banzhaf index in the resulting game is equal to the sum of the probabilistic Banzhaf indices of players 1 and 2 in G.

Task 2 (22 points) *Solution concepts in noncooperative games* **/22 points**

Consider the noncooperative game in normal form with the set $P = \{1, 2\}$ of players, the set $\mathscr{S} = S_1 \times S_2$ of strategy profiles, where $S_i = \{a, b, c, d, e\}$ for $i \in \{1, 2\}$, and the following gain functions:

		Player 2				
		a	b	c	d	e
Player 1	a	$(3, -3)$	$(2, x_2)$	$(1, 2)$	$(1, -1)$	$(-2, 2)$
	b	$(x_1, 0)$	$(1, 1)$	$(3, 3)$	$(1, 0)$	$(-2, -1)$
	c	$(-1, 1)$	$(2, 0)$	$(-1, 1)$	$(0, -1)$	$(1, 1)$
	d	$(3, -2)$	$(-1, 1)$	$(0, 2)$	$(1, 2)$	$(1, 0)$
	e	$(0, 0)$	$(-2, 1)$	$(3, 1)$	$(0, 0)$	$(-1, 1)$

(Please write down all steps of your calculations in each of the following tasks and explain your answers!)

(a) For $x_1 = x_2 = 1$, determine all Nash equilibria in pure strategies and all dominant strategies in this game. For each pair of these Nash equilibria, answer whether one of the Nash equilibria Pareto-dominates the other.

(b) Consider the restriction of the game to the strategy sets $S_1' = \{a, b\}$ and $S_2' = \{a, b\}$. Give a value for x_1 and a value for x_2 such that $((1/3, 2/3), (1/2, 1/2))$ is a Nash equilibrium in mixed strategies in the restricted game.

(c) Consider the restriction of the game to the strategy sets $S_1'' = \{a, e\}$ and $S_2'' = \{a, e\}$. Determine the maxmin strategy of player 1 and the maxmin value of player 1 in this restricted game.

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Task 3 (10 points) *Sequential games, Bayesian games* **/10 points**

Consider the following variant of the Nim game from the lecture, in which two players take turns removing either one, two, or three sticks from a single pile of N sticks. However, they may not take the same number of sticks as the other player in the previous turn. If there is only one stick left, they must take it (even if the player before them took one stick). Player 1 starts. Whoever takes the last stick loses (gain 0), and the other player wins (gain 1).

(a) Draw the complete game tree for the case that $N = 5$ sticks are on the pile at the beginning. In each node, indicate which player's turn it is and how many sticks are left on the pile.

(b) Using your game tree from (a), justify whether there is a winning strategy for player 1 for $N = 5$.

Task 4 (18 points) *Cooperative games* **/18 points**

Consider the following cooperative game $G = (P, v)$ with $P = \{1, 2, 3\}$ and with the following characteristic function:

$$\begin{array}{llll} v(\emptyset) = 0 & v(\{1\}) = 1/2 & v(\{1,2\}) = 2 & v(\{1,2,3\}) = 2 \\ & v(\{2\}) = 0 & v(\{1,3\}) = 1 & \\ & v(\{3\}) = 1 & v(\{2,3\}) = 3/2 & \end{array}$$

(a) Specify a vector that is an imputation for G. Justify your answer.

(b) Show that the core of G is empty.

(c) Give (without justification or derivation) the superadditive cover $G^* = (P, v^*)$ of G. To do this, complete the following table:

C	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v^*(C)$								

(d) Which of the following vectors are in the $1/2$-core of G^*? Check these vectors (without providing a justification).

☐ $(1/2, 1/2, 2)$ ☐ $(1, 1, 1)$ ☐ $(3/2, 1/2, 1/2)$ ☐ $(1/2, 0, 1)$ ☐ $(0, 1.7, 1.3)$

(e) Show that $\mathrm{CoS}(G) \leq 1$.

Task 5 (20 points) *Weighted Voting Games, Shapley Value* **/20 points**

Consider the following weighted voting game with $n = 3$ players where player 3's weight w_3 and the quota q are not specified yet:

$$G = (1, 2, w_3;\, q).$$

(Please prove formally that the properties for your presented games are fulfilled!)

(a) Give values for w_3 and q such that G is convex.

(b) Give values for w_3 and q such that player 1 is a dummy player but player 2 is *not* a dummy player.

(c) Give values for w_3 and q such that player 2 is a veto player but has a Shapley value $\varphi_2(G) < 1$.

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Task 6 (10 points) *Beneficial Splitting* **/10 points**

Consider the following weighted voting game:

$$G = (4, 4, 5; 7)$$

(a) Calculate the Shapley-Shubik index of player 1 in G.

(b) Is $(G, 1, 3)$ a YES-instance of SHAPLEY-SHUBIK-BENEFICIAL-SPLIT? Justify your answer. As a reminder: For a *power index* $\mathbb{PI}$, the problem is defined as follows.

$\mathbb{PI}$-BENEFICIAL-SPLIT	
Given:	A weighted voting game $G = (w_1, \ldots, w_n;\, q)$, a player $i \in \{1, \ldots, n\}$, and an integer $k \geq 2$.
Question:	Is it possible to split i into k new players with positive integer weights $u_1, \ldots, u_k$ satisfying $\sum_{j=1}^{k} u_j = w_i$ so that $$\sum_{j=0}^{k-1} \mathbb{PI}(G_{i \div k}, i + j) > \mathbb{PI}(G, i),$$ where $G_{i \div k} = (w_1, \ldots, w_{i-1}, u_1, \ldots, u_k, w_{i+1}, \ldots, w_n;\, q)$?

Bonus Task (5 bonus points) *Noncooperative Game* **/5 points**

Consider the following noncooperative game in normal form which consists of

- $n \geq 3$ players $P = \{1, \ldots, n\}$,
- the set $\mathcal{S} = S_1 \times \cdots \times S_n$ of strategy profiles with $S_i = \{a, b\}$ for each $i \in P$, and
- the following gain functions g_i for each $i \in P$:

$$g_i(s_1, \ldots, s_n) = \begin{cases} 2 & \text{if } s_i = a \\ 5 \cdot \frac{\|\{j \in P \mid s_j = b\}\|}{\|P\|} & \text{if } s_i = b \end{cases}$$

Which of the following statements are true? Check either "Yes" or "No" at each statement.
You get one point for each correct answer.

☐ Yes ☐ No There exists a dominant strategy for player 1.
☐ Yes ☐ No There exists a Nash equilibrium in pure strategies.
☐ Yes ☐ No For all $n \geq 3$, it is true that $g_1(b, a, \ldots, a) = 5/n$.
☐ Yes ☐ No For $n = 5$, it is true that $g_1(b, b, b, b, a) = 4$.
☐ Yes ☐ No The strategy profile $\vec{s} = (a, \ldots, a)$ is Pareto-optimal.