Argumentation Meets Computational Social Choice

PART I: Preservation of Semantic Properties
Verifying Semantics in Incomplete AFs

PART II: Gradual Acceptance in Argumentation

PART III: Rationalization
Discussion and Outlook

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What is Abstract Argumentation?
The last speaker will start late and should therefore be allowed to exceed her time.
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It is more important to not miss coffee!
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Abstract Argumentation Frameworks

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Abstract Argumentation Frameworks

Diagram: A directed graph with nodes labeled 'a', 'b', and 'c'. Node 'a' points to node 'b', and node 'c' also points to node 'b'.
Abstract Argumentation Frameworks

\[ A \text{, } b, \text{ with: } A \text{, } a, \text{, } b, \text{, } c \] arguments (nodes)
\[ R \text{, } a, \text{, } b, \text{, } c \text{, } b \] attacks (edges)

Abstract Argumentation Frameworks

Argumentation Framework\(^1\)

\[ AF = \langle \mathcal{A}, \mathcal{R} \rangle \text{ with:} \]

\[ \mathcal{A} = \{ a, b, c \} \quad \text{arguments (nodes)} \]

\[ \mathcal{R} = \{ (b, a), (c, b) \} \quad \text{attacks (edges)} \]

Aim: Identify sets of arguments that are acceptable
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\[
C_F: \emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}
\]

conflict-free (CF) no internal attacks
Aim: Identify sets of arguments that are acceptable

conflict-free (CF)  no internal attacks
admissible (AD)  CF & defends all members

AD: ∅, {c}, {a, c}
Aim: Identify sets of arguments that are acceptable

- Conflict-free (CF): no internal attacks
- Admissible (AD): CF & defends all members
- Complete (CP): AD & does not defend non-members

CP extension: \{a, c\}
Aim: Identify sets of arguments that are acceptable

GR extension: \{a, c\}

- conflict-free (CF): no internal attacks
- admissible (AD): CF & defends all members
- complete (CP): AD & does not defend non-members
- grounded (GR): minimal CP
Aim: Identify sets of arguments that are acceptable

PR extension: \{a, c\}

conflict-free (CF) no internal attacks
admissible (AD) CF & defends all members
complete (CP) AD & does not defend non-members
grounded (GR) minimal CP
preferred (PR) inclusion-maximal AD
Aim: Identify sets of arguments that are acceptable

ST extension: \{a, c\}

- conflict-free (CF): no internal attacks
- admissible (AD): CF & defends all members
- complete (CP): AD & does not defend non-members
- grounded (GR): minimal CP
- preferred (PR): inclusion-maximal AD
- stable (ST): CF & attacks all non-members
Stability in Argumentation
Stability in Argumentation May Change over Time

Mr. & Mrs. Smith – Attacking Each Other

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Figure 1: Relations among various semantics for sets of arguments
Figure 2: An argumentation framework

- \{a, c, d\}, \{a, d, f\}, \{a, e, f\}, \{b, d, f\}, and \{b, e, f\} are each conflict-free.
An Example

- \{a, c, d\}, \{a, d, f\}, \{a, e, f\}, \{b, d, f\}, and \{b, e, f\} are each conflict-free
- none of \{a, d, f\}, \{b, d, f\}, and \{b, e, f\} is admissible
  (\{a, d, f\} does not defend \(f\) against \(c\)'s attack and the other two sets do not defend \(b\) against \(a\)'s attack)

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- \{a, c, d\} and \{a, e, f\} are
  - admissible (they defend each of their attacked arguments),
  - complete (they also contain each argument they defend),
  - preferred (adding any other argument to them would violate conflict-freeness), and
  - even stable (they attack each outside argument)
An Example

Figure 2: An argumentation framework

- \{a, c, d\}, \{a, d, f\}, \{a, e, f\}, \{b, d, f\}, and \{b, e, f\} are each **conflict-free**
- none of \{a, d, f\}, \{b, d, f\}, and \{b, e, f\} is **admissible** (\{a, d, f\} does not defend \(f\) against \(c\)'s attack and the other two sets do not defend \(b\) against \(a\)'s attack)
- \{a, c, d\} and \{a, e, f\} are
  - **admissible** (they defend each of their attacked arguments),
  - **complete** (they also contain each argument they defend),
  - **preferred** (adding any other argument to them would violate conflict-freeness), and
  - even **stable** (they attack each outside argument)
- \{a\} is the **grounded** extension
Preservation of Semantic Properties
Aggregation Rules and Axioms

Figure 2: $a \rightarrow b \rightarrow c$ and $d \rightarrow e \rightarrow f$ report this AF

How can we aggregate their individual views so as to reach—as a consensus—a single AF acceptable to the group as a whole?

And how can we preserve useful properties when aggregating argumentation frameworks?

Use COMSOC methods!
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Aggregation Rules and Axioms

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And how can we preserve useful properties when aggregating argumentation frameworks?

⇒ Use COMSOC methods!
What is COMSOC?

This textbook connects three vibrant areas at the interface between economics and computer science: algorithmic game theory, computational social choice, and fair division. It offers a unique interdisciplinary treatment of problems that arise in various applications, focusing on the algorithmic and complexity aspects of these areas.

Part I introduces algorithmic game theory, focusing on both noncooperative and cooperative game theory. Part II introduces computational social choice, focusing on both preference aggregation (voting) and judgment aggregation. Part III introduces fair division, focusing on the division of both a single divisible resource (‘cake-cutting’) and multiple indivisible and unshareable resources (‘multiagent resource allocation’). In all these parts, much weight is given to the algorithmic and complexity aspects of problems among these areas, and the interconnections between these parts are at natural interfaces.
What is COMSOC?
Definition (Chen & Endriss\textsuperscript{a})

Let $\mathcal{A}$ be the set of arguments. An aggregation rule $R$ maps any given profile $\mathcal{P} = (\mathcal{R}_1, \ldots, \mathcal{R}_n)$ of $n$ agents’ individual attack relations on $\mathcal{A}$ to a single attack relation $R(\mathcal{P}) = \mathcal{R}$ on $\mathcal{A}$.

Majority rule:
An attack is in the outcome if and only if a weak majority of agents support it.
Aggregation Rules and Axioms

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In the example above, the outcome under the majority rule is the AF in Figure 2.
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Quota rule with quota $q$: Accept exactly those attacks that have at least $q$ supporters.
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Special cases: majority rule: $q = \lceil n/2 \rceil$, 
Aggregation Rules and Axioms

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Figure 2: and report this AF Figure 3: reports another AF
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Special cases: majority rule: $q = \lceil n/2 \rceil$, unanimity rule: $q = n$, nomination rule: $q = 1$. 
Oligarchic rule: For any coalition $C \neq \emptyset$ of agents, accept exactly those attacks that are supported by all agents in $C$ (neglecting the other agents’ opinions).
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In the example above, if $C = \{\hat{a}, \hat{b}\}$, we get:

Each member of $C$ has veto powers.
Oligarchic rule: For any coalition \( C \neq \emptyset \) of agents, accept exactly those attacks that are supported by all agents in \( C \) (neglecting the other agents’ opinions).

Special cases:

- **unanimity rule** (if \( C = \text{all agents} \))
- **dictatorship** (if \( C = \text{singleton} \))
Chen & Endriss consider the following semantic properties (or axioms).

**Definition**
An aggregation rule $R$ is said to be

(A) *anonymous* if the order of individual attack relations does not matter for $R$;
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**Definition**
An aggregation rule $R$ is said to be

- (A) **anonymous** if the order of individual attack relations does not matter for $R$;
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**Definition**
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(A) _anonymous_ if the order of individual attack relations does not matter for $R$;

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- **(N) neutral** if attacks are treated equally by $R$ whenever they have the same supporters;
- **(I) independent** if $R$ accepting an attack depends solely on its supporters;
- **(M) monotonic** if additional support for an attack accepted by $R$ does never make $R$ reject it;
- **(U) unanimous** if $R$ must accept an attack whenever it is supported by all agents;
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(M) **monotonic** if additional support for an attack accepted by $R$ does never make $R$ reject it;

(U) **unanimous** if $R$ must accept an attack whenever it is supported by all agents;

(G) **grounded** if $R$ can accept an attack only if it is supported by at least one agent.
Theorem (Chen & Endriss\textsuperscript{a})

1. All quota rules and all oligarchic rules are unanimous, grounded, neutral, independent, and monotonic.
Aggregation Rules and Axioms

Theorem (Chen & Endriss\textsuperscript{a})

1. All quota rules and all oligarchic rules are unanimous, grounded, neutral, independent, and monotonic.

2. The quota rules are also anonymous. In fact, the quota rules are the only aggregation rules that satisfy all six axioms (adapting a result by Dietrich & List\textsuperscript{b} from judgment aggregation).


Recall that \( \{a, c, d\} \) and \( \{a, e, f\} \) are stable extensions for the AF in Figure 2.
Recall that \{a, c, d\} and \{a, e, f\} are stable extensions for the AF in Figure 2. Observe that \{a, c, d\} is also stable for the AF in Figure 3 (whereas \{a, e, f\} is not).
Which Semantic Properties Can Be Preserved?

Recall that \{a, c, d\} and \{a, e, f\} are stable extensions for the AF in Figure 2.

Observe that \{a, c, d\} is also stable for the AF in Figure 3 (whereas \{a, e, f\} is not).

**Collective rationality:** A rule aggregating these two argumentation frameworks should output an AF in which \{a, c, d\} still is stable, i.e., *the rule should preserve stability*. Similar so for other semantics.
Which Semantic Properties Can Be Preserved?

**Definition (Chen & Endriss\(^a\))**

For an \( AF = \langle A, \mathcal{R} \rangle \), by **AF-property** (such as stability) we mean the set of all attack relations on \( A \) that satisfy this property.

An aggregation rule \( R \) **preserves an AF-property** \( \Pi \) if for each profile \( \mathcal{P} = (\mathcal{R}_1, \ldots, \mathcal{R}_n) \), if \( \mathcal{R}_i \) is in \( \Pi \) for all \( i \), then so is \( R(\mathcal{P}) \).

---

Which Semantic Properties Can Be Preserved?

Definition (Chen & Endriss\textsuperscript{a})
For an $AF = \langle A, R \rangle$, by \textit{AF-property} (such as stability) we mean the set of all attack relations on $A$ that satisfy this property.

An aggregation rule $R$ \textit{preserves an AF-property} $\Pi$ if for each profile $\mathcal{P} = (R_1, \ldots, R_n)$, if $R_i$ is in $\Pi$ for all $i$, then so is $R(\mathcal{P})$.


Theorem (Chen & Endriss\textsuperscript{a})
For four agents or more, if $\Pi$ is the property of accepting arguments under either the complete, preferred, stable, or grounded semantics then any $R \in N \cap I \cap U \cap G$ preserving $\Pi$ must be a dictatorship.
Theorem (Chen & Endriss$^a$)

1. Every rule $R \in \mathcal{G}$ preserves conflict-freeness.
Which Semantic Properties Can Be Preserved?

Theorem (Chen & Endriss$^a$)

1. Every rule $R \in \mathcal{G}$ preserves conflict-freeness.
2. The nomination rule preserves
   2.1 admissibility for at least four agents (and is the only such rule among those in $A \cap N \cap I \cap M \cap U \cap G$) and
   2.2 stable extensions.
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   2.1 admissibility for at least four agents (and is the only such rule among those in $A \cap N \cap I \cap M \cap U \cap G$) and
   2.2 stable extensions.
3. For at least five agents, each rule in $N \cap I \cap U \cap G$ that preserves grounded extensions must be a dictatorship.
Which Semantic Properties Can Be Preserved?

Theorem (Chen & Endriss\textsuperscript{a})

1. Every rule \( R \in \mathcal{G} \) preserves conflict-freeness.

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3. For at least five agents, each rule in \( N \cap I \cap U \cap G \) that preserves grounded
   extensions must be a dictatorship.

4. For at least four agents, each rule in \( N \cap I \cap U \cap G \) that preserves coherence
   (which says that every preferred extension is stable) must be a dictatorship.
Theorem (Chen & Endriss$^a$)

1. Every rule $R \in G$ preserves conflict-freeness.
2. The nomination rule preserves
   2.1 admissibility for at least four agents (and is the only such rule among those in $A \cap N \cap I \cap M \cap U \cap G$) and
   2.2 stable extensions.
3. For at least five agents, each rule in $N \cap I \cap U \cap G$ that preserves grounded extensions must be a dictatorship.
4. For at least four agents, each rule in $N \cap I \cap U \cap G$ that preserves coherence (which says that every preferred extension is stable) must be a dictatorship.
5. When there are at least as many arguments as there are agents, under each rule in $N \cap I$ that preserves acyclicity or nonemptiness of the grounded extension, at least one agent must have veto powers.
Verifying Semantics in Complete AFs
Computational complexity of verifying extensions
Computational complexity of verifying extensions

**s-VERIFICATION**

**Given:** An argumentation framework $\langle A, R \rangle$ and a subset $S \subseteq A$.

**Question:** Is $S$ an $s$ extension of $AF$?
Computational complexity of verifying extensions

<table>
<thead>
<tr>
<th><strong>s-VERIFICATION</strong></th>
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<tbody>
<tr>
<td><strong>Given:</strong> An argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$ and a subset $S \subseteq \mathcal{A}$.</td>
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<tr>
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</tbody>
</table>

- **PR-VERIFICATION** is coNP-complete.
- For all other semantics considered here, **VERIFICATION** is in P.
Incomplete Knowledge in Abstract Argumentation
Dashed edges represent uncertain attacks
Dashed edges represent *uncertain attacks*
Dashed nodes represent uncertain arguments
Dashed nodes represent uncertain arguments
Possible and Necessary Verification

**s-VERIFICATION:**
Given an AF and arguments S, is S an s extension of AF?
Possible and Necessary Verification

**s-VERIFICATION:**
Given an AF and arguments \( S \), is \( S \) an \( s \) extension of AF?

\[ \Downarrow \]

**s-POSSIBLE-VERIFICATION:**
Given an IAF and arguments \( S \), is \( S \) an \( s \) extension in some completion of IAF?
Possible and Necessary Verification

**s-VERIFICATION:**
Given an AF and arguments $S$, is $S$ an $s$ extension of AF?

**s-POSSIBLE-VERIFICATION:**
Given an IAF and arguments $S$, is $S$ an $s$ extension in some completion of IAF?

**s-NECESSARY-VERIFICATION:**
Given an IAF and arguments $S$, is $S$ is an $s$ extension in all completions of IAF?
Possible and Necessary Winner$^2$

$\varepsilon$-Winner: 
Given an election and a candidate $c$, is $c$ an $\varepsilon$ winner of it?

Possible and Necessary Winner

\(\mathcal{E}\)-Winner: 
Given an election and a candidate \(c\), is \(c\) an \(\mathcal{E}\) winner of it?

\textbf{s-Possible-Winner:} 
Given a partial election and a candidate \(c\), is \(c\) an \(\mathcal{E}\) winner in some of its completions?

---

Possible and Necessary Winner\(^2\)

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Given an election and a candidate \(c\), is \(c\) an \(\mathcal{E}\) winner of it?

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\(\mathcal{s}\)-Necessary-Winner:
Given a partial election and a candidate \(c\), is \(c\) an \(\mathcal{E}\) winner in all of its completions?

Possible and Necessary Winner – Analogues Have also Been Studied in

- **voting** by, e.g., Xia and Conitzer\(^3\), Chevaleyre et al.\(^4\), and Baumeister et al.\(^5\);
- **fair division** by Bouveret et al.\(^6\) and Baumeister et al.\(^7\);
- **algorithmic game theory** by Lang et al.\(^8\); and
- **judgment aggregation** by Baumeister et al.\(^9\).

Verifying Semantics in Incomplete AFs
General Model of Incomplete Argumentation Frameworks
Incomplete Argumentation Frameworks

General Model of Incomplete Argumentation Frameworks
## Complexity Results

<table>
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<th></th>
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