

Argumentation Meets Computational Social Choice

- PART I:** Preservation of Semantic Properties
Verifying Semantics in Incomplete AFs
 - PART II:** Gradual Acceptance in Argumentation
 - PART III:** Rationalization
Discussion and Outlook
-

Dorothea Baumeister, Daniel Neugebauer, and **Jörg Rothe**

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Tutorial 23 at IJCAI-ECAI-18 in Stockholm, Sweden



NRW-FORTSCHRITTSKOLLEG
ONLINE-PARTIZIPATION



What is Abstract Argumentation?

Abstract Argumentation Frameworks

The last speaker will start late
and should therefore be allowed
to exceed her time.

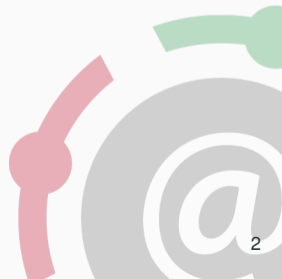


Abstract Argumentation Frameworks

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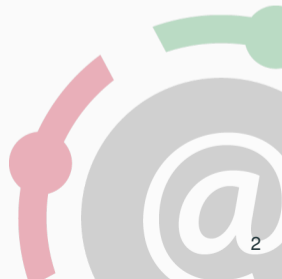
It is more
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Abstract Argumentation Frameworks

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Abstract Argumentation Frameworks

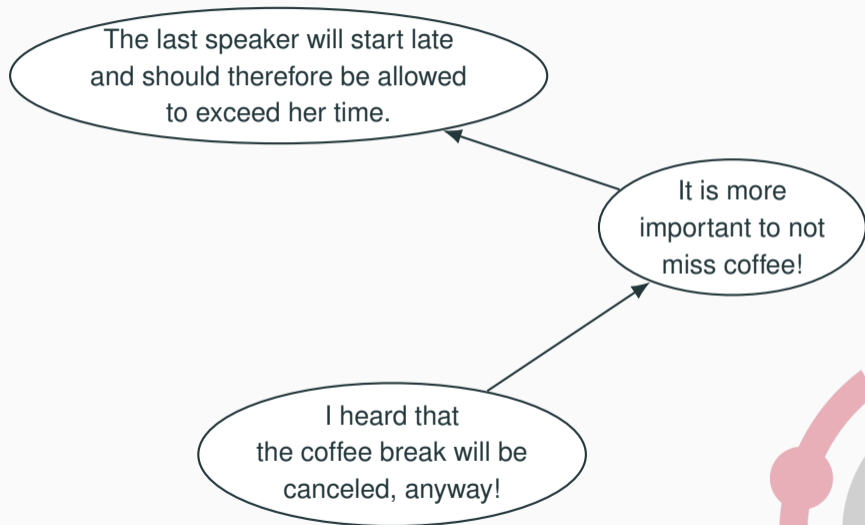
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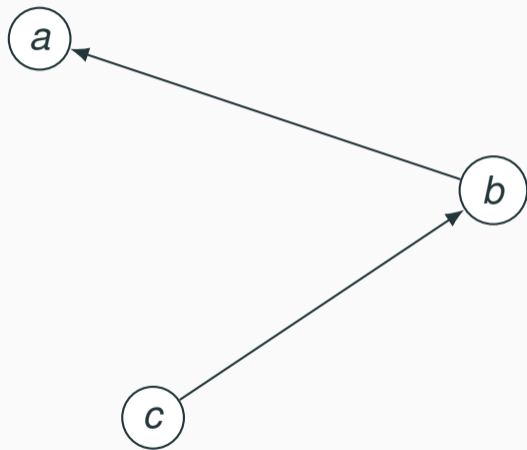
It is more important to not miss coffee!

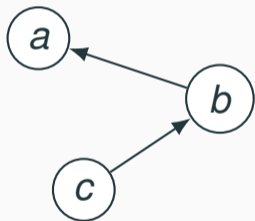
I heard that the coffee break will be canceled, anyway!



Abstract Argumentation Frameworks

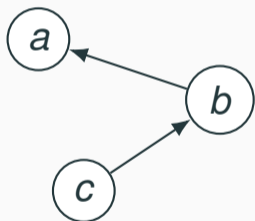






¹P. Dung. "On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n -Person Games". In: *Artificial Intelligence* 77.2 (1995), pp. 321–357.

Argumentation Framework¹



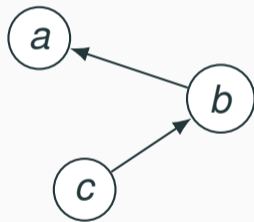
$AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with:

$\mathcal{A} = \{a, b, c\}$ arguments (nodes)

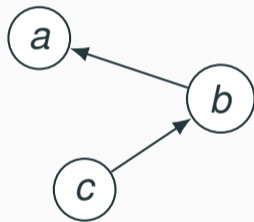
$\mathcal{R} = \{(b, a), (c, b)\}$ attacks (edges)

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Aim: Identify sets of arguments that are acceptable



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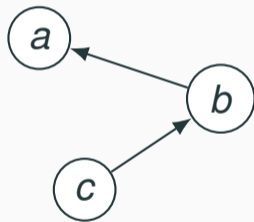


conflict-free (CF) no internal attacks

CF: \emptyset , $\{a\}$, $\{b\}$, $\{c\}$, $\{a, c\}$



Aim: Identify sets of arguments that are acceptable



AD: \emptyset , $\{c\}$, $\{a, c\}$

conflict-free (CF)

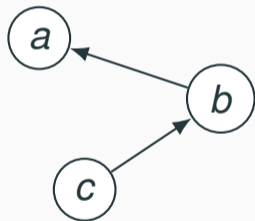
admissible (AD)

no internal attacks

CF & defends all members



Aim: Identify sets of arguments that are acceptable



CP extension: $\{a, c\}$

conflict-free (CF)

admissible (AD)

complete (CP)

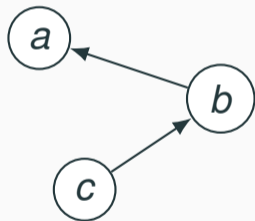
no internal attacks

CF & defends all members

AD & does not defend non-members



Aim: Identify sets of arguments that are acceptable



GR extension: $\{a, c\}$

conflict-free (CF)

admissible (AD)

complete (CP)

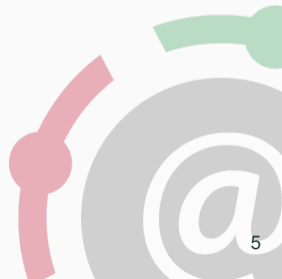
grounded (GR)

no internal attacks

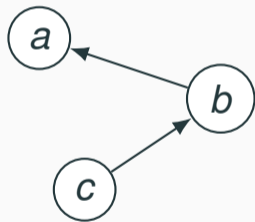
CF & defends all members

AD & does not defend non-members

minimal CP



Aim: Identify sets of arguments that are acceptable



PR extension: $\{a, c\}$

conflict-free (CF)

admissible (AD)

complete (CP)

grounded (GR)

preferred (PR)

no internal attacks

CF & defends all members

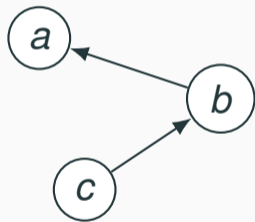
AD & does not defend non-members

minimal CP

inclusion-maximal AD



Aim: Identify sets of arguments that are acceptable



ST extension: $\{a, c\}$

conflict-free (CF)

admissible (AD)

complete (CP)

grounded (GR)

preferred (PR)

stable (ST)

no internal attacks

CF & defends all members

AD & does not defend non-members

minimal CP

inclusion-maximal AD

CF & attacks all non-members

Stability in Argumentation



Mr. & Mrs. Smith – Shooting Scene

© 2005 Twentieth Century Fox

Stability in Argumentation May Change over Time



Mr. & Mrs. Smith – Attacking Each Other

Relations Among Semantics

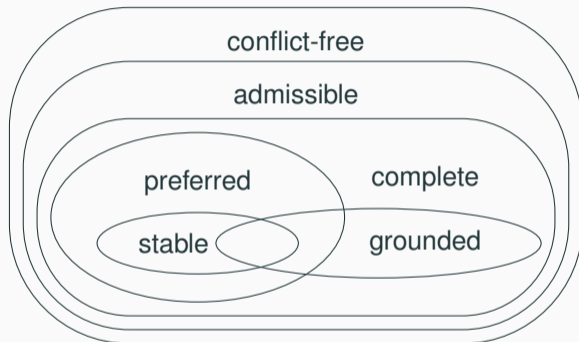
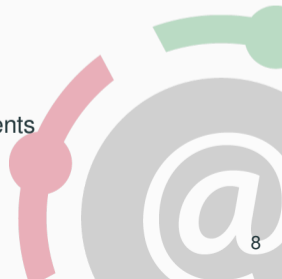


Figure 1: Relations among various semantics for sets of arguments



An Example

- $\{a, c, d\}$, $\{a, d, f\}$, $\{a, e, f\}$, $\{b, d, f\}$, and $\{b, e, f\}$ are each **conflict-free**

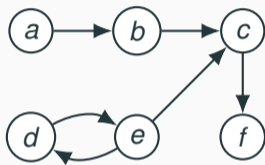


Figure 2: An argumentation framework



An Example

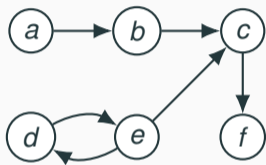


Figure 2: An argumentation framework

- $\{a, c, d\}$, $\{a, d, f\}$, $\{a, e, f\}$, $\{b, d, f\}$, and $\{b, e, f\}$ are each **conflict-free**
- **none of $\{a, d, f\}$, $\{b, d, f\}$, and $\{b, e, f\}$ is admissible** ($\{a, d, f\}$ does not defend f against c 's attack and the other two sets do not defend b against a 's attack)



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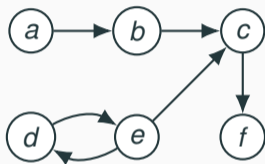


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- $\{a, c, d\}$ and $\{a, e, f\}$ are
 - **admissible** (they defend each of their attacked arguments),
 - **complete** (they also contain each argument they defend),
 - **preferred** (adding any other argument to them would violate conflict-freeness), and
 - **even stable** (they attack each outside argument)

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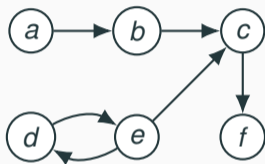
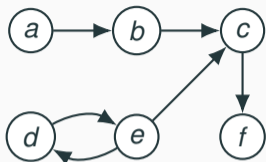


Figure 2: An argumentation framework

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 - **even stable** (they attack each outside argument)
- $\{a\}$ is the **grounded** extension

Preservation of Semantic Properties

Aggregation Rules and Axioms



and



report this AF

Figure 2:  and  report this AF



Aggregation Rules and Axioms

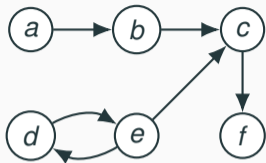


Figure 2:  and  report this AF

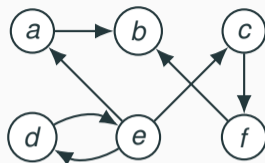


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Aggregation Rules and Axioms

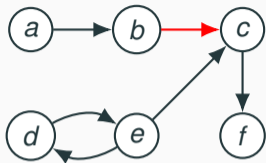


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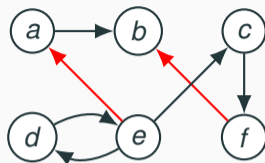


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How can we aggregate their individual views so as to reach—as a consensus—a single AF acceptable to the group as a whole?



Aggregation Rules and Axioms

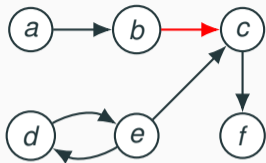


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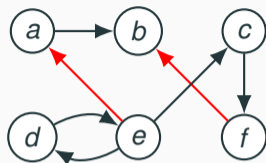


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And how can we preserve useful properties when aggregating argumentation frameworks?

Aggregation Rules and Axioms

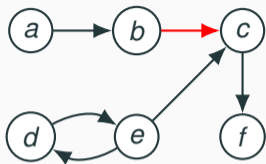


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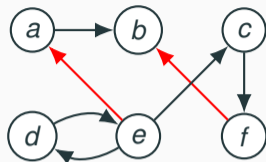


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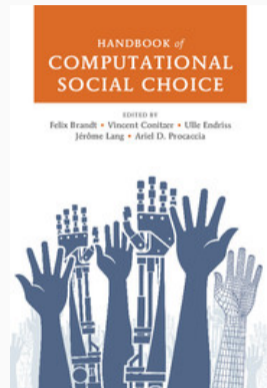
And how can we preserve useful properties when aggregating argumentation frameworks?

⇒ **Use COMSOC methods!**

What is COMSOC?



What is COMSOC?



www.cambridge.org/files/5015/1077/0783/9781107060432AR_final3.pdf

Aggregation Rules and Axioms

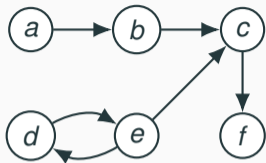


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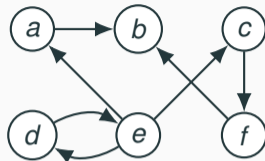


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Definition (Chen & Endriss^a)

Let \mathcal{A} be the set of arguments. An *aggregation rule* R maps any given profile $\mathcal{P} = (\mathcal{R}_1, \dots, \mathcal{R}_n)$ of n agents' individual attack relations on \mathcal{A} to a single attack relation $R(\mathcal{P}) = \mathcal{R}$ on \mathcal{A} .

^aW. Chen and U. Endriss. "Preservation of Semantic Properties during the Aggregation of Abstract Argumentation Frameworks". In: *Proc. TARK'17*. July 2017, pp. 118–129.

Aggregation Rules and Axioms

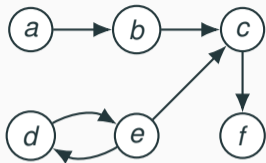


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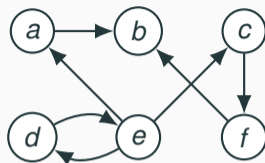


Figure 3:  reports another AF

Majority rule:

An attack is in the outcome if and only if a weak majority of agents support it.

Aggregation Rules and Axioms

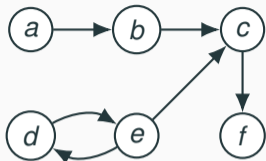


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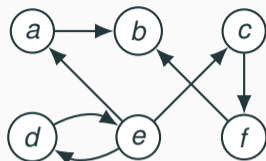


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Aggregation Rules and Axioms

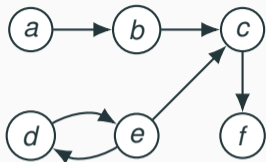


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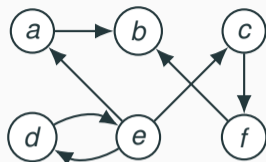


Figure 3:  reports another AF

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Quota rule with quota q : Accept exactly those attacks that have at least q supporters.

Aggregation Rules and Axioms

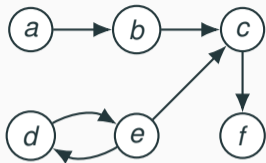


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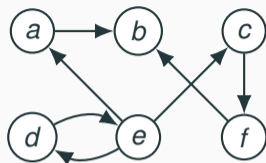


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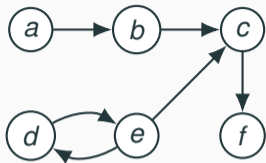


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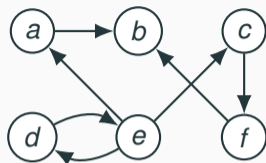


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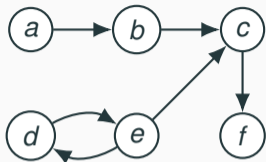


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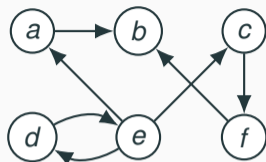


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Aggregation Rules and Axioms

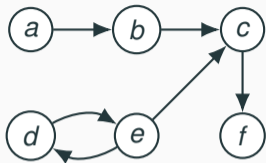


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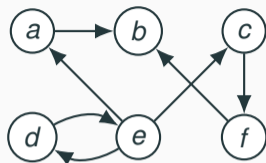


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Oligarchic rule: For any coalition $C \neq \emptyset$ of agents, accept exactly those attacks that are supported by all agents in C (neglecting the other agents' opinions).

Aggregation Rules and Axioms

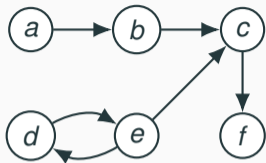


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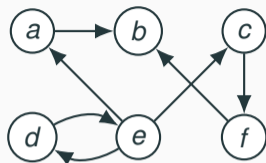
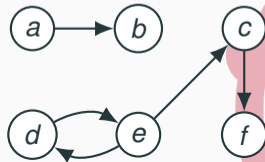


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In the example above, if $C = \left\{ \text{man with glasses}, \text{woman} \right\}$, we get:



Each member of C has *veto powers*.

Aggregation Rules and Axioms

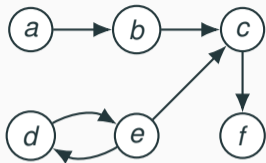


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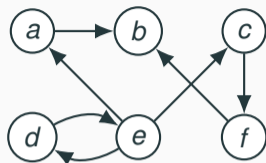


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Special cases:

- **unanimity rule** (if $C =$ all agents)
- **dictatorship** (if $C =$ singleton)

Aggregation Rules and Axioms

Chen & Endriss consider the following semantic properties (or axioms).

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An aggregation rule R is said to be

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- (M) *monotonic* if additional support for an attack accepted by R does never make R reject it;
- (U) *unanimous* if R must accept an attack whenever it is supported by all agents;
- (G) *grounded* if R can accept an attack only if it is supported by at least one agent.

Theorem (Chen & Endriss^a)

1. *All quota rules and all oligarchic rules are unanimous, grounded, neutral, independent, and monotonic.*

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1. *All quota rules and all oligarchic rules are unanimous, grounded, neutral, independent, and monotonic.*
2. *The quota rules are also anonymous.*

*In fact, the quota rules are **the only** aggregation rules that satisfy all six axioms (adapting a result by Dietrich & List^b from judgment aggregation).*

^aW. Chen and U. Endriss. “Preservation of Semantic Properties during the Aggregation of Abstract Argumentation Frameworks”. In: *Proc. TARK'17*. July 2017, pp. 118–129.

^bF. Dietrich and C. List. “Judgment Aggregation by Quota Rules: Majority Voting Generalized”. In: *Journal of Theoretical Politics* 19.4 (2007), pp. 391–424.

Which Semantic Properties Can Be Preserved?

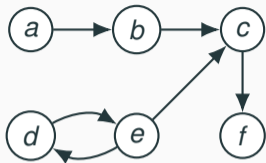


Figure 2:  and  report this AF

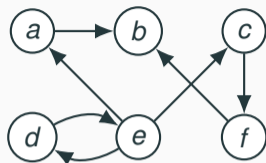


Figure 3:  reports another AF

Recall that $\{a, c, d\}$ and $\{a, e, f\}$ are stable extensions for the AF in Figure 2.

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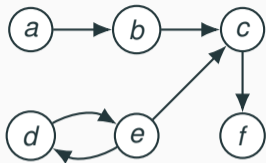


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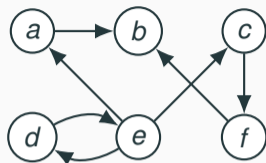


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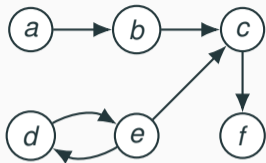


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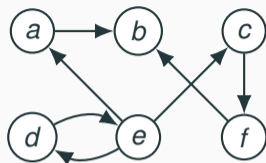


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Collective rationality: A rule aggregating these two argumentation frameworks should output an AF in which $\{a, c, d\}$ still is stable, i.e., *the rule should preserve stability*. Similar so for other semantics.

Which Semantic Properties Can Be Preserved?

Definition (Chen & Endriss^a)

For an $AF = \langle \mathcal{A}, \mathcal{R} \rangle$, by *AF-property* (such as stability) we mean the set of all attack relations on \mathcal{A} that satisfy this property.

An aggregation rule R *preserves an AF-property* Π if for each profile $\mathcal{P} = (\mathcal{R}_1, \dots, \mathcal{R}_n)$, if \mathcal{R}_i is in Π for all i , then so is $R(\mathcal{P})$.

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Theorem (Chen & Endriss^a)

For four agents or more, if Π is the property of accepting arguments under either the complete, preferred, stable, or grounded semantics then any $R \in \mathbb{N} \cap \mathbb{I} \cap \mathbb{U} \cap \mathbb{G}$ preserving Π must be a dictatorship.

Which Semantic Properties Can Be Preserved?

Theorem (Chen & Endriss^a)

1. *Every rule $R \in \mathbb{G}$ preserves conflict-freeness.*

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2. the nomination rule preserves
 - 2.1 admissibility for at least four agents (and is the only such rule among those in $\mathbb{A} \cap \mathbb{N} \cap \mathbb{I} \cap \mathbb{M} \cap \mathbb{U} \cap \mathbb{G}$) and
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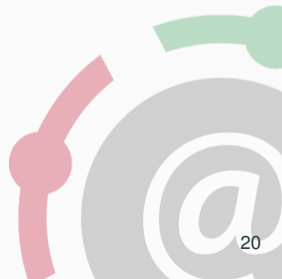
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4. For at least four agents, each rule in $\mathbb{N} \cap \mathbb{I} \cap \mathbb{U} \cap \mathbb{G}$ that preserves coherence (which says that every preferred extension is stable) must be a dictatorship.
5. When there are at least as many arguments as there are agents, under each rule in $\mathbb{N} \cap \mathbb{I}$ that preserves acyclicity or nonemptiness of the grounded extension, at least one agent must have veto powers.

Verifying Semantics in Complete AFs

Computational complexity of verifying extensions

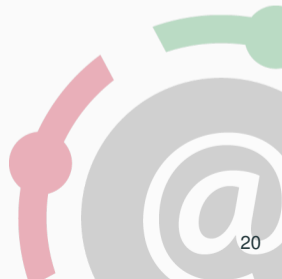


Computational complexity of verifying extensions

s-VERIFICATION

Given: An argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$
and a subset $S \subseteq \mathcal{A}$.

Question: Is S an **s** extension of AF ?



Computational complexity of verifying extensions

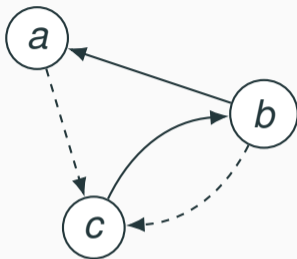
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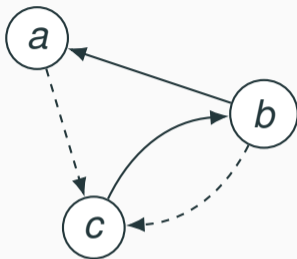
Question: Is S an **s** extension of AF ?

- PR-VERIFICATION is coNP-complete.
- For all other semantics considered here, VERIFICATION is in P.

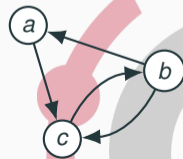
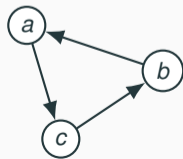
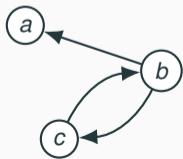
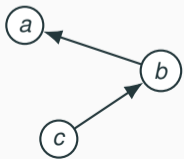
Incomplete Knowledge in Abstract Argumentation

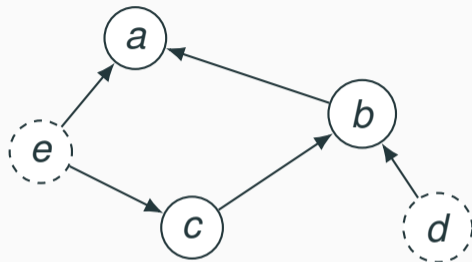


Dashed edges represent **uncertain attacks**



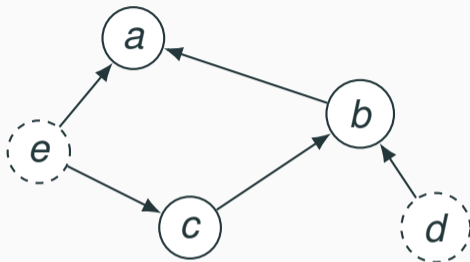
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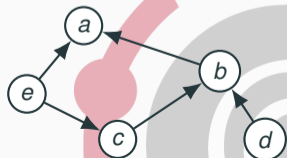
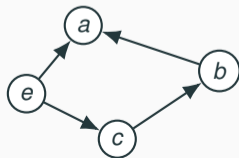
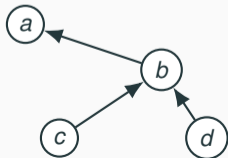
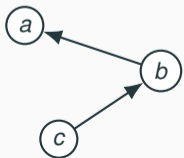


Dashed nodes represent **uncertain arguments**

Argument-incomplete Argumentation Frameworks

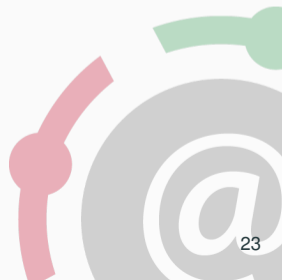


Dashed nodes represent **uncertain arguments**



s-VERIFICATION:

Given an AF and arguments S ,
is S an **s** extension of AF ?



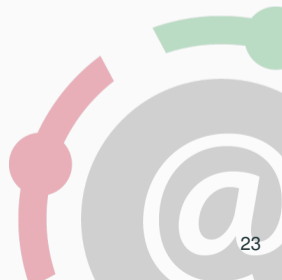
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s-POSSIBLE-VERIFICATION:

Given an IAF and arguments
 S , is S an **s** extension in **some**
completion of IAF ?



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Given an IAF and arguments S ,
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\mathcal{E} -WINNER:

Given an election and a candidate c , is c an \mathcal{E} winner of it?

²K. Konczak and J. Lang. "Voting Procedures with Incomplete Preferences". In: *Proceedings of the Multidisciplinary IJCAI-05 Workshop on Advances in Preference Handling*. 2005, pp. 124–129.

\mathcal{E} -WINNER:

Given an election and a candidate c , is c an \mathcal{E} winner of it?



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Given a partial election and a candidate c , is c an \mathcal{E} winner in **some** of its completions?

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Possible and Necessary Winner – Analogues Have also Been Studied in

- **voting** by, e.g., Xia and Conitzer³, Chevaleyre et al.⁴, and Baumeister et al.⁵;
- **fair division** by Bouveret et al.⁶ and Baumeister et al.⁷;
- **algorithmic game theory** by Lang et al.⁸; and
- **judgment aggregation** by Baumeister et al.⁹.

³L. Xia and V. Conitzer. “Determining Possible and Necessary Winners Given Partial Orders”. In: *Journal of Artificial Intelligence Research* 41 (2011), pp. 25–67.

⁴Y. Chevaleyre et al. “New Candidates Welcome! Possible Winners with respect to the Addition of New Candidates”. In: *Mathematical Social Sciences* 64.1 (2012), pp. 74–88.

⁵D. Baumeister et al. “The Possible Winner Problem with Uncertain Weights”. In: *Proc. ECAI’12*. IOS Press, 2012, pp. 133–138.

⁶S. Bouveret, U. Endriss, and J. Lang. “Fair Division under Ordinal Preferences: Computing Envy-Free Allocations of Indivisible Goods”. In: *Proc. ECAI’10*. IOS Press, 2010, pp. 387–392.

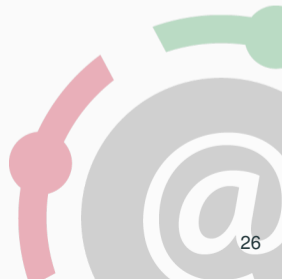
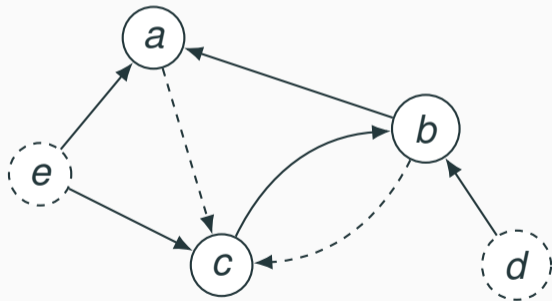
⁷D. Baumeister et al. “Positional Scoring-Based Allocation of Indivisible Goods”. In: *Journal of Autonomous Agents and Multi-Agent Systems* 31.3 (2017), pp. 628–655.

⁸J. Lang et al. “Representing and Solving Hedonic Games with Ordinal Preferences and Thresholds”. In: *Proc. AAMAS’15*. IFAAMAS, 2015.

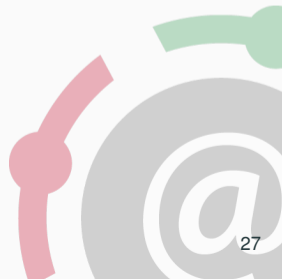
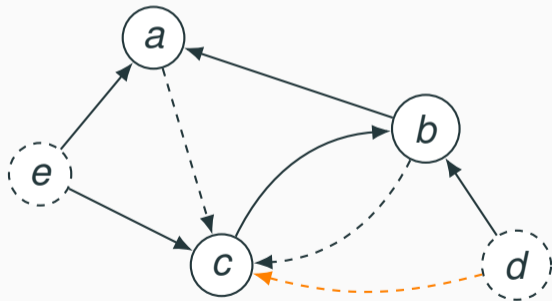
⁹D. Baumeister et al. “Complexity of Manipulation and Bribery in Judgment Aggregation for Uniform Premise-Based Quota Rules”. In: *Mathematical Social Sciences* 76 (2015), pp. 19–30.

Verifying Semantics in Incomplete AFs

General Model of Incomplete Argumentation Frameworks



General Model of Incomplete Argumentation Frameworks



Complexity Results

s	VER	ATTINCPV	ATTINCNV	ARGINCPV	ARGINCNV	INCPV	INCNV
CF	in P [♠]	in P [★]	in P [★]	in P [♦]	in P [♦]	in P [♡]	in P [♡]
AD	in P [♠]	in P [▲]	in P [★]	NP-c. [♦]	in P [♡]	NP-c. [♡]	in P [♡]
ST	in P [♠]	in P [▲]	in P [▲]	NP-c. [♦]	in P [♡]	NP-c. [♡]	in P [♡]
CP	in P [♠]	in P [▲]	in P [▲]	NP-c. [♦]	in P [♡]	NP-c. [♡]	in P [♡]
GR	in P [♠]	in P [▲]	in P [▲]	NP-c. [♦]	in P [♡]	NP-c. [♡]	in P [♡]
PR	coNP-c. [♣]	NP ^{NP} -c. [♡]	coNP-c. [▲]	NP ^{NP} -c. [♡]	coNP-c. [♦]	NP ^{NP} -c. [♡]	coNP-c. [♡]

♠¹⁰, ♣¹¹, ★¹², ▲¹³, ♦¹⁴, ♡¹⁵

¹⁰P. Dung. "On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n -Person Games". In: *Artificial Intelligence* 77.2 (1995), pp. 321–357.

¹¹Y. Dimopoulos and A. Torres. "Graph Theoretical Structures In Logic Programs and Default Theories". In: *Theoretical Computer Science* 170.1 (1996), pp. 209–244.

¹²S. Coste-Marquis et al. "On the Merging of Dung's Argumentation Systems". In: *Artificial Intelligence* 171.10 (2007), pp. 730–753.

¹³D. Baumeister, D. Neugebauer, and J. Rothe. "Verification in Attack-Incomplete Argumentation Frameworks". In: *Proc. ADT'15*. Springer-Verlag LNAI, 2015, pp. 341–358.

¹⁴D. Baumeister, J. Rothe, and H. Schadrack. "Verification in Argument-Incomplete Argumentation Frameworks". In: *Proc. ADT'15*. Springer-Verlag LNAI, 2015, pp. 359–376.

¹⁵D. Baumeister et al. "Complexity of Verification in Incomplete Argumentation Frameworks". In: *Proc. AAAI'18*. AAAI Press, 2018, pp. 1753–1760.