Nice but are they relevant? A political scientist looks at social choice results

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Background

- social choice rules have been studied in somewhat systematic manner for more than two centuries
- over the past half a century the literature grown particularly rapidly
- much of interest in this area is motivated by various flaws of existing voting rules
- yet, very few electoral system reforms have been observed

Why?

Some possible answers:

1. the results tend to be of negative nature
2. the research community is far from unanimous about best systems
3. the nature of the results makes them difficult to “apply”
4. the present system brought you to power, so why change it?

The main points

- Voting rules are instruments with many properties
- Some are mutually compatible, some incompatible
- Not all of the properties are deeded of equal importance
- Patching existing rules may lead to new problems
- Some counterexamples are harder to come by than others
- This pertains the relevance of (negative) results
- Systems can be justified by what we aim at
- Systems may influence opinion patterns
- This also pertains to the relevance of results

What are voting rules used for

- Aggregating opinions.
- Making collective choices.
- Making individual choices
- Settling disagreements.
- Searching for consensus.

Rules make a difference

4 voters	3 voters	2 voters
A	E	D
B	D	C
C	B	B
D	C	E
E	A	A

5 options, 5 winners

Relevance?

- this is just a theoretical example
- with a strong Condorcet winner present, many rules result in it
- even a modicum of consensus increases the coincidence probability of choice rules essentially
- (somewhat contradicting the preceding) most rules have advocates who are not moved by the fact that other rules differ from their favorite

Rationality of rules: what does it mean?

Some views:

- Arrovian view: collective opinions should be similar to the individual ones
- Condorcet requirements
- Consistency
- Choice set invariance
- Monotonicity

Borda's paradox

4 voters	3 voters	2 voters
A	B	C
B	C	B
C	A	A

Borda's points:

- plurality voting results in a bad outcome
- a superior system exists (Borda Count)

Improving Borda Count: Nanson's rule

How does it work? Compute Borda scores and eliminate all candidates with no more than average score. Repeat until the winner is found.

Properties:

- Guarantees Condorcet consistency
- Is nonmonotonic

Nanson's rule is nonmonotonic

30	21	20	12	12	5
C	B	A	B	A	A
A	D	B	A	C	C
D	C	D	C	B	D
B	A	C	D	D	B

The Borda ranking: $A \succ C \succ B \succ D$ with D's score 97 being the only one that does not exceed the average of 150. Recomputing the scores for A, B and C, results in both B and C failing to reach the average of 100. Thus, A wins. Suppose now that those 12 voters who had the ranking $B \succ A \succ C \succ D$ improve A's position, i.e. rank it first, *ceteris paribus*. Now, both B and D are deleted and the winner is C.

Improving plurality rule: plurality runoff

Properties:

- Does not elect Condorcet losers
- Is nonmonotonic

6 voters	*5 voters*	*4 voters*	*2 voters*
A	C	B	B
B	A	C	A
C	B	A	C

Black's system: a synthesis of two ideas

How does it work? Pick the Condorcet winner. If none exists, choose the Borda winner.

Properties:

- Satisfies Cordorcet criteria
- Is monotonic
- Is inconsistent

4 voters	*3 voters*	*3 voters*	*2 voters*	*2 voters*
A	B	A	B	C
B	C	B	C	A
C	A	C	A	B

Some systems and performance criteria

	Criterion								
Voting system	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
Amendment	1	1	1	1	0	0	0	0	0
Copeland	1	1	1	1	1	0	0	0	0
Dodgson	1	0	1	0	1	0	0	0	0
Maximin	1	0	1	1	1	0	0	0	0
Kemeny	1	1	1	1	1	0	0	0	0
Plurality	0	0	1	1	1	1	0	0	1
Borda	0	1	0	1	1	1	0	0	1
Approval	0	0	0	1	0	1	1	0	1
Black	1	1	1	1	1	0	0	0	0
Pl. runoff	0	1	1	0	1	0	0	0	0
Nanson	1	1	1	0	1	0	0	0	0
Hare	0	1	1	0	1	0	0	0	0

Criteria

- a: the Condorcet winner criterion
- b: the Condorcet loser criterion
- c: the strong Condorcet criterion
- d: monotonicity
- e: Pareto
- f: consistency
- g: Chernoff property
- h: independence of irrelevant alternatives
- i: invulnerability to the no-show paradox

Relevance?

- information is “asymmetric”
- failures may be “unlikely” to occur
- behavioral assumptions questionable

More general approach: incompatibility theorems

Examples:

- Arrow
- Gibbard-Satterthwaite
- Moulin
- Young

Relevance?

- Arrow: IIA often violated with impunity
- Gibbard-Satterthwaite: computational complexity issues
- Moulin-Young: Condorcet winners often ignored
- how often do we get into trouble?

Things may be open for interpretation: Kemeny's rule

Consider a partition of a set N of individuals with preference profile ϕ into two separate sets of individuals N_1 and N_2 with corresponding profiles ϕ_1 and ϕ_2 over A and assume that $f(\phi_1 \cap \phi_2) \neq \emptyset$. The social choice function f is consistent iff $f(\phi_1 \cap \phi_2) = f(\phi)$, for all partitionings of the set of individuals.
The same definition can be applied to social preference functions. F is consistent iff $F(\phi_1) \cap F(\phi_2) \neq \emptyset$ implies that $F(\phi_1) \cap F(\phi_2) = F(\phi)$.
As a choice function Kemeny's rule is inconsistent (Fishburn). As a preference function it is consistent.

Spatial representation

The individuals are supposed to be endowed with complete and transitive preference relations $\succeq$ over all point pairs in the space W. These relations are, moreover, assumed to be representable by utility functions in the usual way, that is

$$x \succeq y \Leftrightarrow u(x) \geq u(y), \forall x, y \in W$$

In strong spatial models the individual i's evaluations of alternatives are assumed to be related to a distance measure d_i defined over the space. Moreover, each individual i is assumed to have an ideal point x_i in the space so that

$$x \succeq y \Leftrightarrow d_i(x, x_i) \leq d_i(y, x_i), \forall x, y \in W$$

The voter support system

- is based on voter and candidate interviews or questionnaires
- determines the subjects' stand on a variety of political issues
- (sometimes) asks the subject to determine the weight of each issue
- defines a distance measure between stands on each issue
- determines the proximity of candidates to the voter

My problem

Why is it that the closest candidate rarely gets my vote? (And I'm not alone in this: a large majority of Finns feel the same way.)
Possible explanations:

- I may have different metric in computing the closest candidates
- I may have other issues and criteria in mind than those considered by the system
- I may exhibit Ostrogorski's or related aggregation paradox

Indirect or direct democracy

Ostrogorski's paradox:

issue	*issue 1*	*issue 2*	*issue 3*	*the voter votes for*
voter A	X	X	Y	X
voter B	X	Y	X	X
voter C	Y	X	X	X
voter D	Y	Y	Y	Y
voter E	Y	Y	Y	Y
winner	Y	Y	Y	?

Reinterpretation

- criterion A: relevant educational background
- criterion B: political experience
- criterion C: negotiation skills
- criterion D: substance expertise
- criterion E: relevant political connections

Suppose that the criterion-wise preference is formed on the basis of which alternative is better on more issues than the other. If all issues and criteria are deemed importance, the decision of which candidate the individual should vote is ambiguous: the row-column aggregation with the majority principle suggests X, but the column-row aggregation with the same principle yields Y.

Exam paradox reinterpreted

Example

Nermuth. One of two competitors, X, is located at the following distance from the voter's ideal point in a multi-dimensional space. The score of X on each criterion is simply the arithmetic mean of its distances rounded to the nearest integer and in the case of a tie down to the nearest integer.

issue	1	2	3	4	average	score
criterion 1	1	1	2	2	1.5	1
criterion 2	1	1	2	2	1.5	1
criterion 3	1	1	2	2	1.5	1
criterion 4	2	2	3	3	2.5	2
criterion 5	2	2	3	3	2.5	2

Exam paradox cont'd

Example

X's competitor Y, in turn, is located in the space as follows.

issue	1	2	3	4	average	score
criterion 1	1	1	1	1	1.0	1
criterion 2	1	1	1	1	1.0	1
criterion 3	1	1	2	3	1.75	2
criterion 4	1	1	2	3	1.75	2
criterion 5	1	2	1	2	1.75	2

Anscombe's paradox

Example

issue	*issue 1*	*issue 2*	*issue 3*
voter 1	Y	Y	X
voter 2	X	X	X
voter 3	X	Y	Y
voter 4	Y	X	Y
voter 5	Y	X	Y

Ostrogorski vs. Anscombe

Example

voter	*issue 1*	*issue 2*	*issue 3*	*majority alternative*
1	X	X	Y	X
2	X	Y	X	X
3	Y	X	X	X
4	Y	Y	Y	Y
5	Y	Y	X	Y

Simpson's paradox before Simpson

Cohen and Nagel (1934):

Example

death rate per 100.000	*New York*	*Richmond*
sub-population 1	179	162
sub-population 2	560	332
total death rate	187	226

System choice in simple settings

1. A satisfies the criterion, while B doesn't, i.e. there are profiles where B violates the criterion, but such profiles do not exist for B.
2. in every profile where A violates the criterion, also B does, but not vice versa.
3. in *practically all profiles* where A violates the criterion, also B does, but not vice versa ("A dominates B almost everywhere").
4. in a plausible probability model B violates the criterion with higher probability than A.
5. in those political cultures that we are interested in, B violates the criterion with higher frequency than A.

The role of culture

- impartial culture: each ranking is drawn from uniform probability distribution over all rankings
- impartial anonymous culture: all profiles (i.e. distributions of voters over preference rankings) equally likely
- unipolar cultures
- bipolar cultures

Lessons from probability and simulation studies

- cultures make a difference (Condorcet cycles, Condorcet efficiencies, discrepancies of choices)
- none of the cultures mimics “reality”
- IC is useful in studying the proximity of intuitions underlying various procedures

What makes some incompatibilities particularly dramatic?

The fact that they involve intuitively plausible, “natural” or “obvious” desiderata. The more plausible etc. the more dramatic is the incompatibility.

Theorem

Moulin, Pérez: all Condorcet extensions are vulnerable to the no-show paradox.

Example

26%	47%	2%	25%
A	B	B	C
B	C	C	A
C	A	A	B

Some “difficult” counterexamples: Black

Black’ procedure is vulnerable to the no-show paradox, indeed, to the strong version thereof.

1 voter	1 voter	1 voter	1 voter	1 voter
D	E	C	D	E
E	A	D	E	B
A	C	E	B	A
B	B	A	C	D
C	D	B	A	C

Here D is the Condorcet winner and, hence, is elected by Black.
Suppose now that the right-most voter abstains. Then the Condorcet winner disappears and E emerges as the Borda winner. It is thus elected by Black. E is the first-ranked alternative of the abstainer.

Another difficult one: Nanson

5 voters	5 voters	6 voters	1 voter	2 voters
A	B	C	C	C
B	C	A	B	B
D	D	D	A	D
C	A	B	D	A

Here Nanson's method results in B.
If one of the right-most two voters abstain, C – their favorite – wins. Again the strong version of no-show paradox appears.
The twin paradox occurs whenever a voter is better off if one or several individuals, with identical preferences to those of the voter, abstain. Here we have an instance of the twin paradox as well: if there is only one CBDA voter, C wins. If he is joined by another, B wins.

Dodgson

42 voters	26 voters	21 voters	11 voters
B	A	E	E
A	E	D	A
C	C	B	B
D	B	A	D
E	D	C	C

A here is closest to becoming the Condorcet winner, i.e. it is the Dodgson winner.
Now take 20 out the 21 voter group out. Then B becomes the Condorcet and, thus, Dodgson winner. B is preferred to A by the abstainers, demonstrating vulnerability to the no-show paradox. Adding those 20 twins back to retrieve the original profile shows that Dodgson is also vulnerable to the twin paradox.

Young

Again inspired by and adapted from Pérez (2001) and Moulin (1988):

11	10	10	2	2	2	1	1
B	E	A	E	E	C	D	A
A	C	C	C	D	B	C	B
D	B	D	D	C	A	B	D
E	D	B	B	B	D	A	E
C	A	E	A	A	E	E	C

In this profile E is elected (needs only 12 removals). Add now 10 voters with ranking EDABC. This makes D the Condorcet winner. Hence, the 10 added voters are better of abstaining. Indeed we have an instance of the strong version of no-show paradox. Obviously, twins are not always welcome here.

Simpson-Kramer

5 voters	4 voters	3 voters	3 voters	4 voters
D	B	A	A	C
B	C	D	D	A
C	A	C	B	B
A	D	B	C	D

The outranking matrix is:

	A	B	C	D	row min
A	-	10	6	14	6
B	9	-	12	8	8
C	13	7	-	8	7
D	5	11	11	-	5

B is elected. With the 4 CABD voters abstaining, the outcome is A. With only 1 CABD voter added to the 15-voter profile, A is still elected. If one then adds 3 “twins” of the CABD voter, one ends up with B being elected. Hence twins are not welcome.

Is the Condorcet condition plausible?

Starting profile:

7 voters	*4 voters*
A	B
B	C
C	A

Add a Condorcet paradox profile:

4 voters	*4 voters*	*4 voters*
A	B	C
C	A	B
B	C	A

to get a new Condorcet winner.

Learning from proofs

Some proofs are (almost) constructive, i.e. tell us how to generate paradoxes. Pérez uses the following auxiliary result. Let $p(x, y) =$ the no. of voters preferring x to y.

Theorem

For any Condorcet extension which is invulnerable to no-show paradox, for any situation (X, p) and for any pair x, z of alternatives, if $p(x, z) < min_{y \in X} p(z, y)$, then $x \notin f(X, p)$.

In words, the antecedence says that the minimum support for z is larger than the no. of votes x receives in comparison with z. The consequence says that then x is not elected (provided that the f is Condorcet and invulnerable).

Learning . . ., cont'd

The theorem is then used to construct an example.

5	4	3	3
t	y	x	x
y	z	t	t
z	x	z	y
x	t	y	z

Applying the Theorem to pairs $(z, y), (\mathbf{y}, \mathbf{t}), (t, x)$ it turns out that only x is chosen.
Add now 4 voters with ranking *zxyt* and apply Theorem to pairs $(t, x), (\mathbf{x}, \mathbf{z}), (z, y)$ to find that y is chosen.

What do we aim at?

Possible consensus states:

- consensus about everything, i.e. first, second, etc.
- consensus about the winner
- majority consensus about first rank
- majority consensus about Condorcet winner
- ...

How far are we?

Possible distance measures:

- inversion metric (Kemeny)
- discrete metric
- ...

Upshot

We have (hopefully) seen that:

- system-criterion pairs give “asymmetric” information
- only important criteria ought to be focused upon
- the likelihood of encountering problems varies with the culture
- some counterexamples are much harder to find than others
- systems should be evaluated in terms of what they are used for
- systems may “cause” preferences

What is called for is (much) more work on structural properties of problematic profiles.