An Empirical Study of Borda Manipulation

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Motivation

- One of the “last” open questions in manipulation
  - What is the computational complexity of manipulating Borda?
- Computational social choice can borrow heuristics from scheduling

Borda

- Score based voting rule
  - ith candidate gets score m-i
- Due to Llull (13thC), Jean Charles de Borda (1770), ..
- Used in anger
  - Eurovision, Robocup, MVP in baseball, several

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Manipulating Borda

- [Xia, Conitzer, Procaccia EC 2010]

"The exact complexity of the problem [coalition manipulation with unweighted votes] is now known with respect to almost all of the prominent voting rules, with the glaring exception of Borda"

- Some evidence to suggest it may be suspectible
  - Theoretical, empirical, historical

Manipulating Borda

- Theoretical
  - Problem has an FPTAS, greedy heuristic needs at most one extra manipulator
- Empirical
  - Strategic voting was seen in 1991 presidential candidate elections for the Republic of Kiribati
- Historical
  - Borda appears to have recognized its manipulabilty: *"My scheme is intended only for honest men"*

Manipulating Borda

- Recast as bin packing
  - Bins=candidates
  - Weights=scores
  - Put max. score in bin you want to win, other bins need to be no bigger
  - Each bin contains same number of items

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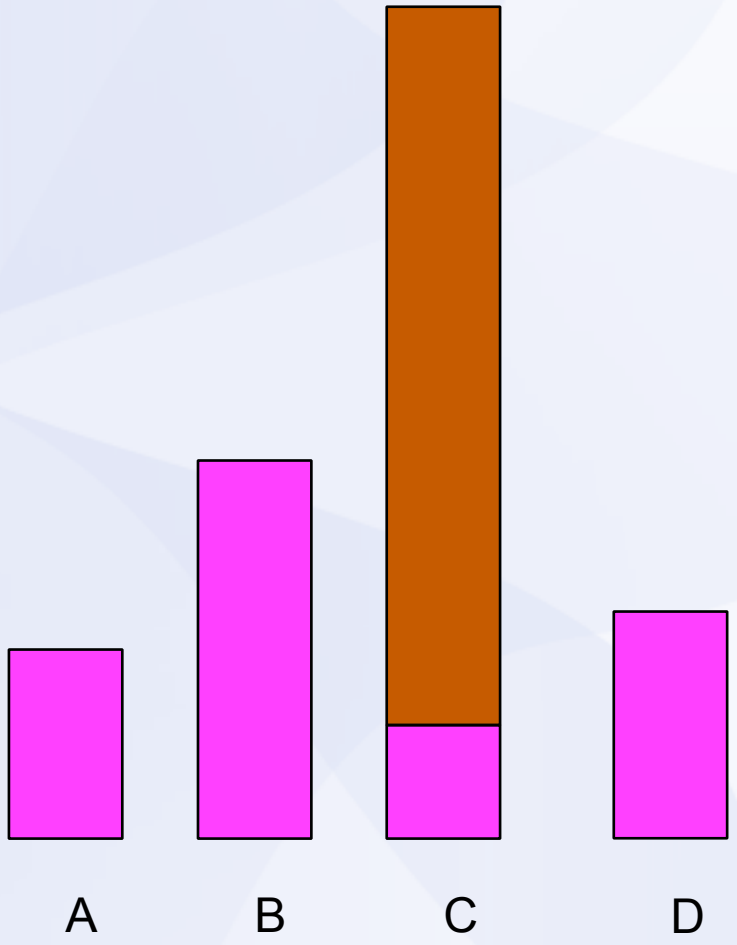

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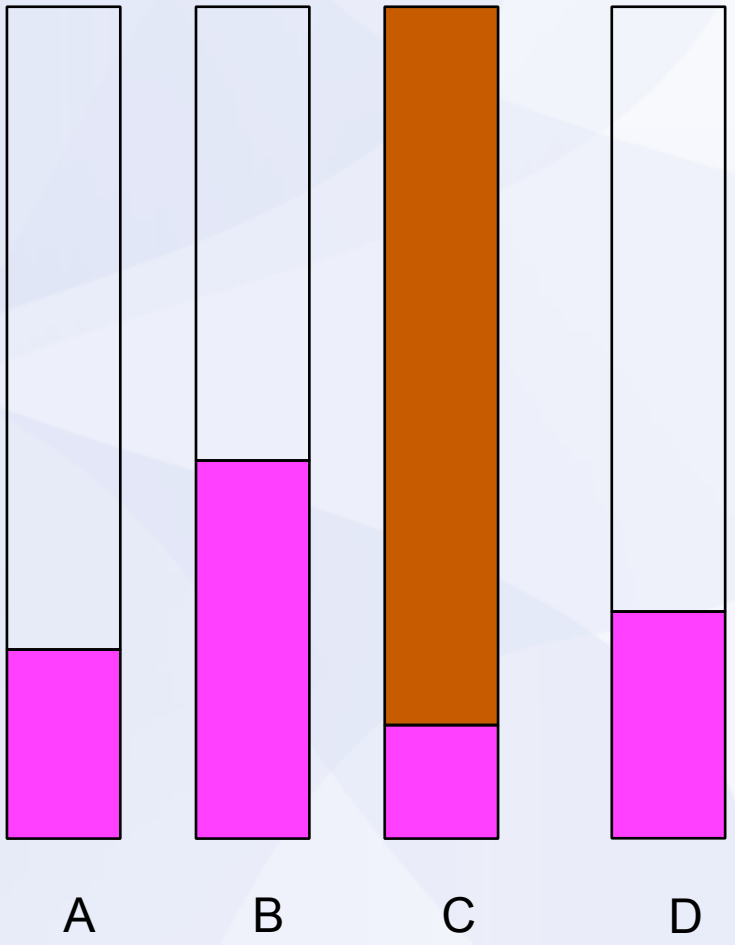
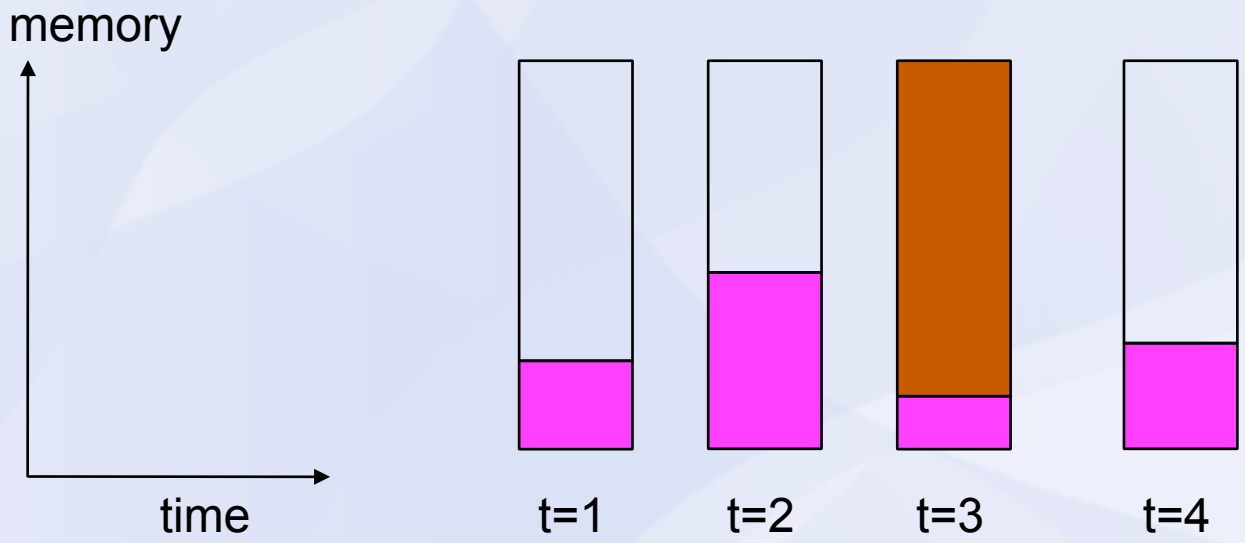


“Layer” constraints irrelevant!

- Thm: if there exists a bin packing containing k copies of 0,..,m-1 then there exists a bin packing in which each layer contains 0,..,m-1
  - Proof: Complex induction on number of rows (=manipulators). Calls upon Hall's matching theorem

Borda manipulation=bin packing



- Compute manipulation with bin packing heuristics
  - Constraint that bins contains equal number of items makes it equivalent to multiprocessor scheduling with unit execution time and varying memory footprint

Existing GREEDY heuristic

- [Zuckerman, Procaccia & Rosenschein SODA 2008]
  - Manipulators fill bins in turn, putting largest weight in smallest bin
  - Uses at most one extra manipulator than optimum

First new heuristic

We don't have to consider manipulators in turn (see previous theorem)

HEUR1

Order n(m-1) scores

m-1,m-1,..,m-1,m-2,m-2,..

Repeat

- Put largest score in bin with most space

Similar to [Krause et al, JACM 1975] for multiprocessor scheduling

Theoretical properties

- Good news

  Thm: Infinite class of problems on which HEUR1 finds optimal 2-manipulation on which GREEDY finds 3-manipulation

- Bad news

  Thm: Infinite class of problems on which GREEDY finds optimal manipulation but HEUR1 requires O(n) extra manipulators

Second new heuristic

We don't have to consider manipulators in turn but we should consider #items in each bin

HEUR2

Order n(m-1) scores

m-1,m-1,..,m-1,m-2,m-2,..

Repeat

- Put largest (possible) score in bin where space available/items missing is largest

Theoretical properties

- Good news

  Thm: Infinite class of problems on which HEUR2 finds optimal 2-manipulation on which GREEDY finds 3-manipulation

- Bad news

  Thm: Exist problems on which GREEDY finds optimal manipulation but HEUR2 does not

Empirical performance

- Same experimental setup as [Walsh, ECAI 2010]
  - Uniform random elections (IC)
  - Urn model (Poly-Eggenberger)
- Found optimal manipulation as CSP problem
  - Remember: not known if this is NP-hard!

Empirical performance

- Success rate at finding optimal manipulation
  - Random elections

GREEDY: 75%, HEUR1: 83%, HEUR2: 99%

HEUR2 never beaten by GREEDY

  - Urn elections

GREEDY: 74%, HEUR1: 42%, HEUR2: 99.7%

HEUR2 beaten in 1 out of >30,000 problems by GREEDY

Conclusions

- Borda appears easy to manipulate
  - Simple greedy heuristics often find optimal manipulations
  - It pays not to construct manipulation voter by voter
- Open questions
  - What is the exact computational complexity of Borda manipulation?
  - Are these results useful for other scoring rules?