Parameterized Control Complexity in Bucklin Voting and in Fallback Voting

Gábor Erdélyi[1] Michael R. Fellows[2]

[1]Institut für Informatik, Heinrich-Heine-Universität Düsseldorf, Germany

[2]Charles Darwin University, Australia

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Outline

Voting

- Preference aggregation and collective decision-making.
- Political science, economics, social choice theory, and operations research.
- In computer science:
  - artificial intelligence (multiagent systems)
  - planning
  - similarity search
  - design of ranking algorithms

Dealing with NP-Hardness

Worst-case complexity vs.

- approximation algorithms
- algorithms that are always efficient although not always correct
- algorithms that are always correct, but not always efficient
- average-case complexity
- parameterized complexity

Parameterized Complexity

- Fixed-parameter tractability: Membership in FPT.
- Fixed-parameter intractability:

$$FPT = W[0] \subseteq W[1] \subseteq W[2] \ldots$$

- Reductions from Dominating Set.

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  - to make a desired candidate win (constructive),
  - to prevent a despised candidate from winning (destructive).

How to Affect the Outcome of an Election

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- The Bad Guy can have two different intentions:
  - to make a desired candidate win (constructive),
  - to prevent a despised candidate from winning (destructive).
- Computational barrier to prevent cheating in elections.
  - **Control:** The Chair modifies the election's structure.
  - **Bribery:** (Not considered here) An external agent bribes a group of voters.
  - **Manipulation:** (Not considered here) An evil coalition of voters strategically change their votes.

Elections & Voting Systems

- Set of candidates and multiset of voters:
  - $C = \{c_1, \ldots, c_m\}$,
  - $V = \{v_1, \ldots, v_n\}$.
- Voter preferences over C can be represented as
  - preference lists (rankings),
  - approval/disapproval vectors.
- Voting rule aggregates the preferences and outputs the set of winners:
  - unique-winner model,
  - nonunique-winner model.

Control

- Candidate Control:
  - Adding candidates
  - Deleting candidates
  - Partition of candidates
    - With runoff
    - Without runoff

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- Tie Handling:
  - Ties eliminate (TE)
  - Ties promote (TP)

Example

Name: Constructive Control by Adding Voters.

Instance: An election $(C, V \cup W)$, a designated candidate $c \in C$, and a positive integer k.

Parameter: k.

Question: Is it possible to choose a subset $W' \subseteq W$ with $\|W'\| \leq k$ such that c is the unique winner of the resulting $(C, V \cup W')$?

Control

- Candidate Control:
  - Adding candidates
  - Deleting candidates
- Voter Control:
  - Adding voters
  - Deleting voters

Contrast

Table

Number of resistances, immunities, and vulnerabilities to the 22 common control types.

Number of	*AV*	*Llull*	*Copeland*	*Plurality*	*BV*	*SP-AV*	*FV*
resistances	4	14	15	16	≥ 18	19	≥ 19
immunities	9	0	0	0	0	0	0
vulnerabilities	9	8	7	6	≤ 4	3	≤ 3

Fallback Voting

- Proposed by Brams and Sanver (2009).
- Line between acceptable and inacceptable candidates:

$$\{c_4, c_1\} \mid \{c_2, c_3, c_5, c_6\}.$$

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- In addition each voter has a preference ranking, a tie-free linear ordering of all approved candidates:

$$c_4 > c_1 | \{c_2, c_3, c_5, c_6\}.$$

Example for Fallback Voting

Example

Preferences:

- $v_1 = a > b > c > \{d, e\}$
- $v_2 = a > b > \{c, d, e\}$
- $v_3 = c > \{a, b, d, e\}$
- $v_4 = d > e > b > \{a, c\}$
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Votes:

- $a \;\; b \;\; c \mid \{d, e\}$
- $a \;\; b \mid \{c, d, e\}$
- $c \mid \{a, b, d, e\}$
- $d \;\; e \;\; b \mid \{a, c\}$
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	a	b	c	d	e
Level 1 score	2	0	2	1	0

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	a	b	c	d	e
Level 2 score	3	2	2	1	1

Bucklin Voting

- Each voter has a tie-free linear ordering of all candidates:

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- $M_t = \lfloor n/2 \rfloor + 1$

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- $score^i_{(C,V)}(c)$ = number of voters who rank c on level i or higher.
- $M_t = \lfloor n/2 \rfloor + 1$
- $score_B(c) = min\{i \mid score^i_{(C,V)}(c) \geq M_t\}$
- Winner: The candidate with the lowest Bucklin score.

Example for Bucklin Voting

Example

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- $v_2 = a > b > c > e > d$
- $v_3 = c > b > a > d > e$
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Scores:

- $score_B(a) = 2$
- $score_B(b) = 2$
- $score_B(c) = 3$
- $score_B(d) = 4$
- $score_B(e) = 4$

$$score^2_{(C,V)}(a) = 3 < 4 = score^2_{(C,V)}(b)$$

Previous Results

Theorem

Control by	Fallback Voting		Bucklin	
	Constructive	*Destructive*	*Constructive*	*Destructive*
Adding a Limited Number of Candidates	NP-*complete*	NP-*complete*	NP-*complete*	NP-*complete*
Deleting Candidates	NP-*complete*	NP-*complete*	NP-*complete*	NP-*complete*
Adding Voters	NP-*complete*	*P*	NP-*complete*	*P*
Deleting Voters	NP-*complete*	*P*	NP-*complete*	*P*

Results

Theorem

Control by	*Fallback Voting*		*Bucklin*	
	Constructive	*Destructive*	*Constructive*	*Destructive*
Adding a Limited Number of Candidates	*W[2]-hard*	*W[2]-hard*	*W[2]-hard*	*W[2]-hard*
Deleting Candidates	*W[2]-hard*	*W[2]-hard*	*W[2]-hard*	*W[2]-hard*
Adding Voters	*W[2]-hard*	*FPT*	*W[2]-hard*	*FPT*
Deleting Voters	*W[2]-hard*	*FPT*	*W[2]-hard*	*FPT*

Conclusions and Open Questions

- The problems remain hard for the natural parameterization.
- What is the complexity if parameterized by the amount of action and the number of voters/candidates?
- Partition cases are still open.

Thank you very much!