Approximate Judgement Aggregation
(for the case of the doctrinal paradox)

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Doctrinal Paradox

Research Question: Approximate Aggregation

Approximate Aggregation Results
  - for The Doctrinal Paradox
  - for Other Agendas
  - for a Class of Agendas

Conclusion
Suppose a defendant is accused in court of murder. In order to prove his guiltiness, one should convince the judge of two independent issues:

(A) The defendant killed the victim

(B) The defendant is sane

Conviction is defined to be the conjunction of the first two issues

\((A \land B)\) The defendant is guilty.
## Doctrinal Paradox (Unpacking the court/Kornhauser and Sager 1986)

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$A \land B$</th>
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</thead>
<tbody>
<tr>
<td>(Killed)</td>
<td>(Sane)</td>
<td>(Guilty)</td>
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- $A \land B$ inconsistent

$$
\begin{array}{ccc}
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
0 & 0 & 0 \\
\end{array}
$$
Doctrinal Paradox (Unpacking the court/Kornhauser and Sager 1986)

<table>
<thead>
<tr>
<th>$A$ (Killed)</th>
<th>$B$ (Sane)</th>
<th>$A \land B$ (Guilty)</th>
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Agenda
Doctrinal Paradox (Unpacking the court/ Kornhauser and Sager 1986)

<table>
<thead>
<tr>
<th></th>
<th>$A$ (Killed)</th>
<th>$B$ (Sane)</th>
<th>$A \land B$ (Guilty)</th>
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</thead>
<tbody>
<tr>
<td>Judge 1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Judge 2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Judge 3</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Majority</td>
<td>1</td>
<td>1</td>
<td>0</td>
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</table>
A profile \( X \in \{0, 1\}^{n \times m} \) where:

- \( n \) : Number of voters
- \( m = 3 \) : Number of issues

The opinion of the \( i^{th} \) voter on the 2\( ^{nd} \) issue:

\[
\begin{bmatrix}
X_1^1 & X_2^1 & X_3^1 \\
\vdots & \vdots & \vdots \\
X_i^1 & X_i^2 & X_i^3 \\
\vdots & \vdots & \vdots \\
X_n^1 & X_n^2 & X_n^3
\end{bmatrix}
\]
Notations

\[ X_1 \quad X_2 \quad X_3 = X_1 \land X_2 \]
\[ \vdots \quad \vdots \quad \vdots \]
\[ X_i^{1} \quad X_i^{2} \quad X_i^{3} = X_i^{1} \land X_i^{2} \]
\[ \vdots \quad \vdots \quad \vdots \]
\[ X_n^{1} \quad X_n^{2} \quad X_n^{3} = X_n^{1} \land X_n^{2} \]

The \( i^{th} \) row \( X_i \) represents the **consistent** opinion of the \( i^{th} \) voter.
Notations

A profile $X \in \{0, 1\}^{n \times m}$

$F \left[ \begin{array}{cccc}
X_1^1 & X_1^2 & \ldots & X_1^m \\
\vdots & \vdots & \ddots & \vdots \\
X_i^1 & X_i^2 & \ldots & X_i^m \\
\vdots & \vdots & \ddots & \vdots \\
X_n^1 & X_n^2 & \ldots & X_n^m
\end{array} \right] = (a_1, a_2, a_3)$

The $j^{th}$ column $X^j$ represents the opinions of all voters on the $j^{th}$ issue
Notations

A profile $X \in \{0, 1\}^{n \times m}$ ($n$: Number of voters, $m$: Number of issues)

$$F \left( \begin{array}{ccc}
X_1^1 & X_1^2 & X_1^3 = X_1^1 \wedge X_1^2 \\
\vdots & \vdots & \vdots \\
X_i^1 & X_i^2 & X_i^3 = X_i^1 \wedge X_i^2 \\
\vdots & \vdots & \vdots \\
X_n^1 & X_n^2 & X_n^3 = X_n^1 \wedge X_n^2
\end{array} \right) = (a_1, a_2, a_3)$$

An aggregation mechanism returns for every profile an aggregated opinion

$$F : \left\{ \{0, 1\}^m \right\}^n \rightarrow \{0, 1\}^m$$
Notations

\[ F \left( \begin{array}{ccc} X_1^1 & X_1^2 & X_i^3 = X_i^1 \land X_i^2 \\ \vdots & \vdots & \vdots \\ X_i^1 & X_i^2 & X_i^3 = X_i^1 \land X_i^2 \\ \vdots & \vdots & \vdots \\ X_n^1 & X_n^2 & X_n^3 = X_n^1 \land X_n^2 \end{array} \right) = (a_1, a_2, a_3) \]

Definition (Consistency)

\( F \) is **consistent** if it returns a consistent result whenever all voters voted consistently

\[ a_3 = a_1 \land a_2 \]
Notations

A profile \( X \in \{0, 1\}^{n \times m} \) (\( n \): Number of voters, \( m = 3 \): Number of issues)

\[
F \begin{pmatrix}
X_1^1 & X_1^2 & X_1^3 = X_1^1 \land X_1^2 \\
\vdots & \vdots & \vdots \\
X_i^1 & X_i^2 & X_i^3 = X_i^1 \land X_i^2 \\
\vdots & \vdots & \vdots \\
X_n^1 & X_n^2 & X_n^3 = X_n^1 \land X_n^2
\end{pmatrix} = (a_1, a_2, a_3)
\]

Definition (Independence)

\( F \) is \textbf{independent} if the aggregated opinion of the \( j^{th} \) issue depends solely on the votes for the \( j^{th} \) issue.
Notations

A profile $X \in \{0, 1\}^{n \times m}$ ($n$: Number of voters, $m = 3$: Number of issues)

$$F\begin{pmatrix}
X_1^1 & X_1^2 & X_i^3 = X_i^1 \land X_i^2 \\
\vdots & \vdots & \vdots \\
X_i^1 & X_i^2 & X_i^3 = X_i^1 \land X_i^2 \\
\vdots & \vdots & \vdots \\
X_n^1 & X_n^2 & X_n^3 = X_n^1 \land X_n^2
\end{pmatrix} = (a_1, a_2, a_3)$$

**Definition (Independence)**

$F$ is **independent** if the aggregated opinion of the $j^{th}$ issue depends solely on the votes for the $j^{th}$ issue
Notations

\[ F \left( \begin{array}{ccc}
X_1^1 & X_1^2 & X_i^3 = X_i^1 \land X_i^2 \\
\vdots & \vdots & \vdots \\
X_i^1 & X_i^2 & X_i^3 = X_i^1 \land X_i^2 \\
\vdots & \vdots & \vdots \\
X_n^1 & X_n^2 & X_n^3 = X_n^1 \land X_n^2
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Definition (Independence)

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Notations

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X_i^1 & X_i^2 & X_i^3 = X_i^1 \land X_i^2 \\
\vdots & \vdots & \vdots \\
X_n^1 & X_n^2 & X_n^3 = X_n^1 \land X_n^2 
\end{array} \right) = (a_1, a_2, a_3) \]

**Definition (Independence)**

\( F \) is **independent** if the aggregated opinion of the \( j^{\text{th}} \) issue depends solely on the votes for the \( j^{\text{th}} \) issue.
Aggregation Mechanism - Examples

\[ X_1^1 \quad X_2^1 \quad \cdots \quad X_3^1 = X_1^1 \land X_2^1 \]
\[ \vdots \quad \vdots \quad \cdots \quad \vdots \]
\[ X_i^1 \quad X_i^2 \quad \cdots \quad X_i^3 = X_i^1 \land X_i^2 \]
\[ \vdots \quad \vdots \quad \cdots \quad \vdots \]
\[ X_n^1 \quad X_n^2 \quad \cdots \quad X_n^3 = X_n^1 \land X_n^2 \]

Independence:
Consistency:

Are there any other consistent and independent aggregation mechanisms?
Aggregation Mechanism - Examples

\[
\begin{align*}
X_1^1 & = X_1^1 \land X_1^2 \\
\vdots & \quad \vdots \\
X_i^1 & = X_i^1 \land X_i^2 \\
\vdots & \quad \vdots \\
X_n^1 & = X_n^1 \land X_n^2
\end{align*}
\]

Issue-wise Majority : \( Maj(X^1) \quad Maj(X^2) \quad Maj(X^3) \)

Independence: ✓
Consistency: X
Aggregation Mechanism - Examples

\[
\begin{array}{ccc}
X_1^1 & X_1^2 & X_1^3 = X_1^1 \land X_1^2 \\
\vdots & \vdots & \vdots \\
X_i^1 & X_i^2 & X_i^3 = X_i^1 \land X_i^2 \\
\vdots & \vdots & \vdots \\
X_n^1 & X_n^2 & X_n^3 = X_n^1 \land X_n^2 \\
\end{array}
\]

Premise Majority: \( Maj(X^1) \quad Maj(X^2) \quad Maj(X^1) \land Maj(X^2) \)

Independence: \( \times \)
Consistency: \( \checkmark \)
Aggregation Mechanism - Examples

\[
\begin{align*}
X_1^1 & = X_1^2 & X_3^1 & = X_1^1 \land X_1^2 \\
\vdots & & \vdots & \\
X_i^1 & = X_i^2 & X_i^3 & = X_i^1 \land X_i^2 \\
\vdots & & \vdots & \\
X_n^1 & = X_n^2 & X_n^3 & = X_n^1 \land X_n^2
\end{align*}
\]

Constant:

| $0$ | $g(X^2)$ | $0$ |

Independence: ✔
Consistency: ✔
### Aggregation Mechanism - Examples

<table>
<thead>
<tr>
<th>X_1^1</th>
<th>X_1^2</th>
<th>X_1^3 = X_1^1 \land X_1^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>X_i^1</td>
<td>X_i^2</td>
<td>X_i^3 = X_i^1 \land X_i^2</td>
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<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>X_n^1</td>
<td>X_n^2</td>
<td>X_n^3 = X_n^1 \land X_n^2</td>
</tr>
</tbody>
</table>

| Oligarchy: | \land X^1 | \land X^2 | \land X^3 |

- **Independence:** ✓
- **Consistency:** ✓
Aggregation Mechanism - Examples

\[
\begin{align*}
X_1^1 &\quad X_2^1 &\quad X_3^1 = X_1^1 \land X_2^1 \\
\vdots &\quad \vdots &\quad \vdots \\
X_i^1 &\quad X_i^2 &\quad X_i^3 = X_i^1 \land X_i^2 \\
\vdots &\quad \vdots &\quad \vdots \\
X_n^1 &\quad X_n^2 &\quad X_n^3 = X_n^1 \land X_n^2
\end{align*}
\]

Independence:
Consistency:

Are there any other consistent and independent aggregation mechanisms?
Definition (Oligarchy)

An oligarchy of $S$ returns 1 iff all the members of $S$ voted 1.

$$u_S(\bar{x}) = \bigwedge_{i \in S} x_i$$
Theorem

Let $F$ be an independent and consistent aggregation mechanism for $\langle A, B, A \land B \rangle$. Then there exists three boolean functions $f, g, h : \{0, 1\}^n \to \{0, 1\}$ s.t. $F(X) = \langle f(X^1), g(X^2), h(X^3) \rangle$ and

- $f = h \equiv 0$
- or $g = h \equiv 0$
- or $f = g = h$ and it is an oligarchy.

This theorem is a direct corollary from a series of works in the more general framework of aggregation. (E.g., Nehring&Puppe 2007, Holzman&Dokow 2008)
Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \land B \rangle$. Then
**Theorem**

Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \land B \rangle$. Then

**Definition ($\delta$-consistent)**

$F$ is $\delta$-consistent if the following test fails with probability at most $\delta$:

Choose a consistent profile $X$ uniformly at random. Check whether $F(X)$ is a consistent opinion.
Theorem

Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \land B \rangle$. Then

Definition ($\delta$-independent)

$F$ is $\delta$-independent if the following test fails with probability at most $\delta$:

- Choose a consistent profile $X$ uniformly at random.
- Choose an issue $j$ uniformly at random.
- Choose a random consistent profile $Y$ s.t. $X^j = Y^j$.
- Check whether $(F(X))^j$ equals $(F(Y))^j$. 


Research Question

**Theorem**

Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \land B \rangle$. Then

Notice that

$0$-consistency $\equiv$ Consistency

$0$-independence $\equiv$ Independence
Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \land B \rangle$. Then

Notice that

$0$-consistency $\equiv$ Consistency
$0$-independence $\equiv$ Independence

Moreover, for $\delta < C \cdot 4^{-n} \approx \frac{1}{\text{Number of profiles}}$,

$\delta$-consistency $\equiv$ Consistency
$\delta$-independence $\equiv$ Independence
Theorem

Let $\delta > \exp(n, \epsilon)$

Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \land B \rangle$.

Then there exists an independent and consistent aggregation mechanism $G$ that agrees with $F$ on at least $1 - \epsilon$ of the profiles.

Notice that

0-consistency $\equiv$ Consistency
0-independence $\equiv$ Independence

Moreover, for $\delta < C \cdot 4^{-n} \approx \frac{1}{\text{Number of profiles}}$

$\delta$-consistency $\equiv$ Consistency
$\delta$-independence $\equiv$ Independence
Theorem

Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \land B \rangle$. Then there exists an independent and consistent aggregation mechanism $G$ that agrees with $F$ on at least $1 - \epsilon$ of the profiles.

The other direction is trivial.

Theorem

Let $F$ and $G$ be two aggregation mechanisms for $\langle A, B, A \land B \rangle$ such that:

- $G$ is independent and consistent
- $F$ and $G$ agree on at least $1 - \epsilon$ of the profiles

Then $F$ is $\epsilon$-independent and $6\epsilon$-consistent.
Main result for $\langle A, B, A \land B \rangle$

**Theorem**

For any $\epsilon > 0$ and $\delta = \text{poly}(\epsilon, n)$:  
$(\delta \approx C \cdot n^{-2} \epsilon^5)$

Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \land B \rangle$.

Then there exists an independent and consistent aggregation mechanism $G$ that agrees with $F$ on at least $1 - \epsilon$ of the profiles.
Techniques - How did we get this result?

- Restricting ourself to independent mechanisms.
- Applying an (agenda independent) technique to extend the result to $\delta$-independence and $\delta$-consistency.
Techniques - How did we get this result?

Given an independent $\delta$-consistent aggregation mechanism $F = \langle f, g, h \rangle$
Given an independent \( \delta \)-consistent aggregation mechanism \( F = \langle f, g, h \rangle \)

**Definition (Influence (Banzhaf Power Index))**

The **influence** of the \( i^{\text{th}} \) voter on \( f \) is the probability he can change the result by changing his vote.

\[
I_i(f) = \Pr[f(x) \neq f(x \oplus e_i)]
\]

**Definition (Ignorability)**

The **ignorability** of the \( i^{\text{th}} \) voter on \( f \) is the probability \( f \) returns 1 although \( i \) voted 0.

\[
P_i(f) = \Pr[f(x) = 1|x_i = 0]
\]
Given an independent $\delta$-consistent aggregation mechanism $F = \langle f, g, h \rangle$

We show that

- $f$ is an oligarchy iff

$$\forall i : I_i(f) P_i(f) = 0$$
Given an independent $\delta$-consistent aggregation mechanism $F = \langle f, g, h \rangle$

We show that

- $f$ is an oligarchy iff

$$\forall i : I_i(f)P_i(f) = 0$$

- $$\forall i : I_i(f)P_i(g) \leq 4\delta$$
Given an independent $\delta$-consistent aggregation mechanism $F = \langle f, g, h \rangle$

We show that

- $f$ is an oligarchy iff

  $$\forall i : I_i(f)P_i(f) = 0$$

- $$\forall i : I_i(f)P_i(g) \leq 4\delta$$

Let $u$ be the oligarchy of the voters with small ignorability (either $P_i(f)$ or $P_i(g)$). Then,

- $f$ and $g$ are close to $u$
- $F$ is close to $\langle u, u, u \rangle$. 

Techniques - How did we get this result?
Agenda

- Doctrinal Paradox
- Research Question: Approximate Aggregation
- Approximate Aggregation Results
  - for The Doctrinal Paradox
  - for Other Agendas
  - for a Class of Agendas
- Conclusion
Doctrinal Paradox

Research Question: Approximate Aggregation

Approximate Aggregation Results
- for The Doctrinal Paradox
- for Other Agendas
  - Preference Agenda
  - XOR Agenda $\langle A, B, A \oplus B \rangle$
- for a Class of Agendas

Conclusion
Other Agendas - Preference Aggregation

<table>
<thead>
<tr>
<th>$a &gt; b$</th>
<th>$b &gt; c$</th>
<th>$c &gt; a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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Other Agendas - Preference Aggregation

<table>
<thead>
<tr>
<th></th>
<th>a &gt; b</th>
<th>b &gt; c</th>
<th>c &gt; a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voter 1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Voter 2</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Voter 3</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Majority</td>
<td>1</td>
<td>1</td>
<td>1</td>
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Theorem (Condorcet Paradox)

*Pair-wise majority might lead to inconsistent outcome.*
Theorem (Condorcet Paradox)
Pair-wise majority might lead to inconsistent outcome.

Theorem (Arrow’s Theorem 1950)
So is any other non-dictatorial aggregation mechanism that satisfies independence and Pareto.
### Theorem (Condorcet Paradox)

*Pair-wise majority might lead to inconsistent outcome.*

### Theorem (Arrow’s Theorem 1950)

*So is any other non-dictatorial aggregation mechanism that satisfies independence and Pareto.*

### Theorem (Kalai 2002, Mossel 2009)

*For any $\epsilon > 0$:
Let $F$ be an independent, $K\epsilon$-consistent (and balanced) preference aggregation mechanism. Then there exists an independent and consistent aggregation mechanism $G$ (i.e., dictatorship) that agrees with $F$ on at least $1 - \epsilon$ of the profiles.*
Other Agendas - $\langle A, B, A \oplus B \rangle$

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$A \oplus B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>1</td>
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<td>0 $\leftarrow$ inconsistent</td>
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<td>0 $\leftarrow$ inconsistent</td>
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Other Agendas - $\langle A, B, A \oplus B \rangle$

**Theorem**

For any $\epsilon > 0$ and $\delta = \text{poly}(\epsilon, n)$:  
$(\delta = C \cdot \epsilon)$

Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $\langle A, B, A \oplus B \rangle$.

Then there exists an independent and consistent aggregation mechanism $G$ that agrees with $F$ on at least $1 - \epsilon$ of the profiles.
Techniques - How did we get this result?

- Restricting ourself to independent mechanisms.
- Applying an (agenda independent) technique to extend the result to $\delta$-independence and $\delta$-consistency.
Techniques - How did we get this result?

Given an independent $\delta$-consistent aggregation mechanism $F = \langle f, g, h \rangle$
We describe $f, g, h$ using Fourier representation and prove that

$$1 - 2\delta = \sum_{\chi} \hat{f}(\chi) \hat{g}(\chi) \hat{h}(\chi)$$

when

- The summation is over all functions $\chi$ s.t. $\langle \chi, \chi, \chi \rangle$ is consistent
- $\left| \hat{f}(\chi) \right|$ equals $1 - 2d$ for $d$ being the distance between $f$ and $\chi$.

in order to get that $F$ is ‘close to’ $\langle \chi, \chi, \chi \rangle$. 
Main result

Theorem

For any $\epsilon > 0$, $m, n \geq 1$, and $\delta = \text{poly} \left( \frac{1}{n}, \epsilon, m \right)$:
Let $X$ be a premise-conclusion agenda over $m$ issues in which each issue is either a premise, or a conclusion of at most two premises.
Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $X$.
Then there exists an independent and consistent aggregation mechanism $G$ that agrees with $F$ on at least $1 - \epsilon$ of the profiles.
Main result

**Theorem**

For any $\epsilon > 0$, $m, n \geq 1$, and $\delta = \text{poly} \left( \frac{1}{n}, \epsilon, m \right)$:

Let $X$ be a premise-conclusion agenda over $m$ issues in which each issue is either a premise, or a conclusion of at most two premises.

Let $F$ be a $\delta$-independent and $\delta$-consistent aggregation mechanism for $X$.

Then there exists an independent and consistent aggregation mechanism $G$ that agrees with $F$ on at least $1 - \epsilon$ of the profiles.

For instance:

\[
\langle A, B, A \oplus B \rangle \\
\langle A, B, A \land B, A \lor B \rangle \\
\langle A, B, C, A \land B \lor C \rangle \\
\langle A, B, C, A \land B, B \oplus C, A \land C \rangle \\
\langle A \land B, B \land C, C \land A \rangle
\]
Theorem

For any $\epsilon > 0$, $m, n \geq 1$, and $\delta = \text{poly}\left(\frac{1}{n}, \epsilon, m\right)$:

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Theorem

For any $\epsilon > 0$, $m, n \geq 1$, and $\delta = poly \left( \frac{1}{n}, \epsilon, m \right)$:

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Main result

Theorem

For any $\epsilon > 0$, $m, n \geq 1$, and $\delta = \text{poly} \left( \frac{1}{n}, \epsilon, m \right)$:

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Technique: • $\land$ and $\oplus$ represent all boolean functions of two arguments.

• Induction over the number of issues.
We defined the question of approximate aggregation.
Summary

- We defined the question of approximate aggregation.
- We proved approximate aggregation theorems for \( \langle A, B, A \land B \rangle \) and \( \langle A, B, A \oplus B \rangle \).

Open question: Find an agenda and an aggregation mechanism that is \( \delta \)-independent and \( \delta \)-consistent but is far from any independent consistent aggregation mechanism.
• We defined the question of approximate aggregation.
• We proved approximate aggregation theorems for $\langle A, B, A \land B \rangle$ and $\langle A, B, A \oplus B \rangle$.
• We proved approximate aggregation theorems for a class of premise conclusion agendas.

Open question:
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  - Find an agenda and an aggregation mechanism that is $\delta$-independent and $\delta$-consistent but is far from any independent consistent aggregation mechanism.

Thank You
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Please write me any comments/questions/suggestions you have.