Complexity of Winner Determination and Strategic Manipulation in Judgment Aggregation

Ulle Endriss Umberto Grandi Daniele Porello

Institute for Logic, Language and Computation
University of Amsterdam

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Discursive Dilemma

JA was developed to generalise and study paradoxical situations that arise when a collective judgment has to be made on a set of correlated propositions.

Discursive Dilemma

	p	$p \to q$	q
Agent 1:	Yes	Yes	Yes
Agent 2:	No	Yes	No
Agent 3:	Yes	No	No
Majority:	Yes	Yes	No

Each individual is rational (i.e., has a consistent judgment) but the majority is contradictory!

Kornhauser and Sager. Unpacking the court. Yale Law Journal, 1986.

Impossibility Results

This paradoxical situation can be generalised to an impossibility result:

Theorem [List and Pettit, 2002]

If the agenda contains at least two atoms and a conjunction $p, q, p \wedge q$ *then there exists no aggregation procedure satisfying anonymity, systematicity and collective rationality.*

Many other results on this line defining several agenda properties:

Theorem [Nehring and Puppe, 2006]

There exists anonymous, systematic, monotonic and collectively rational procedures iff the agenda satisfies the median property.

List and Pettit. Aggregating sets of judgments. Economics and Philosophy, 2002.

Nehring and Puppe, The structure of strategy-proof social choice I. JET 2006

Basic Definitions

A set N of individuals expressing judgments on a set of correlated propositions:

Definition

An agenda is a finite subset of propositional formulas $\Phi \subseteq \mathcal{L}_{PS}$ closed under complementation and not containing double negations.

A judgment set on an agenda Φ is a subset $J \subseteq \Phi$.

Denote with $J(\Phi)$ the set of all consistent and complete judgment sets over Φ:

Definition

An aggregation procedure for agenda Φ and a set N of n individuals is a function $F : J(\Phi)^n \to 2^\Phi$.

First Study of Complexity: Safety of the Agenda

Definition

An agenda Φ is safe with respect to a class of aggregation procedures $\mathcal{F}_\Phi$ if every procedure in $\mathcal{F}_\Phi$ has a consistent outcome in every profile.

How difficult is to check safety?

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How difficult is to check safety?

We reduce this problem to checking properties of the inconsistent subsets of an agenda (e.g., median property, $\mathrm{SSMP}\ldots$) and we prove that:

Complexity Result

$\mathrm{SAFETY}[\mathcal{F}]$ *is Π_2^p-complete for several classes $\mathcal{F}$ of procedures.*

Endriss, Grandi and Porello. Complexity of JA: Safety of the Agenda. AAMAS-2010.

Strategic Manipulation in JA

Manipulation in voting theory: *A player can manipulate a voting rule when there exists a situation in which misrepresent her preferences result in an outcome that she prefers to the current one.*

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We need a notion of individual preference in JA:

Hamming Distance (following Dietrich and List)

If J, J' are two complete and complement-free judgment sets, the Hamming distance $H(J, J')$ is the number of positive formulas on which they differ.

Manipulability

A JA procedure F is said to be manipulable by agent i at profile $\mathbf{J} = (J_1, \ldots, J_i, \ldots, J_n)$ if there exist an alternative judgment set $J_i' \in J(\Phi)$ such that $H(J_i, F(J_i', \mathbf{J}_{-i})) < H(J_i, F(\mathbf{J}))$.

Dietrich and List, Strategy-proof judgment aggregation. Economics and Philosophy, 2007.

Premise-based procedure

Definition (PBP)

If $\Phi = \Phi_p \uplus \Phi_c$ is divided into premises and conclusions. The premise-based procedure aggregates a profile $\mathbf{J}$ to a judgment set $\Delta \cup \Gamma$ where:

- $\Phi_p \supseteq \Delta = \{\varphi \in \Phi_p \mid \#\{i \mid \varphi \in J_i\} > \frac{n}{2}\}$
- $\Phi_c \supseteq \Gamma = \{\varphi \in \Phi_c \mid \Delta \models \varphi\}$

We want PBP to be consistent and complete $\Longrightarrow$ Agenda closed under propositional symbols, Φ_p as the set of literals

Kornhauser and Sager. The one and the many... California Law Review, 1993.

Dietrich and Mongin. The premiss-based approach to JA. JET, 2010.

Results I

WinDet(F)
Instance: Agenda Φ, profile $\mathbf{J}$, formula $\varphi \in \Phi$.
Question: Is φ an element of $F(\mathbf{J})$?

Theorem (easy proof)
WinDet(PBP) *is in P.*

Manipulable(F)
Instance: Agenda Φ, judgment set J_i, partial profile $\mathbf{J}_{-i}$.
Question: Is there a J_i' s.t. $H(J_i, F(J_i', \mathbf{J}_{-i})) < H(J_i, F(J_i, \mathbf{J}_{-i}))$?

Theorem (reduction from SAT)
Manipulability(PBP) *is NP-complete.*

Distance-based procedure

Definition (DBP)

Given an agenda Φ, the distance-based procedure DBP is the function mapping each profile $\mathbf{J} = (J_1, \ldots, J_n)$ to the following set of judgment sets:

$$\mathrm{DBP}(\mathbf{J}) = \arg \min_{J \in J(\Phi)} \sum_{i=1}^{n} H(J, J_i)$$

A collective outcome under this procedure minimises the amount of disagreements with the individual judgment sets.

Pigozzi. Belief merging and the discursive dilemma. Synthese, 2006.

Results II

$\textsc{WinDet}^\star(\text{DBP})$		
Instance:	Agenda Φ, profile $\mathbf{J} \in J(\Phi)^n$, formula $\varphi \in \Phi$, $K \in \mathbb{N}$.	
Question:	Is there a $J^\star \in J(\Phi)$ with $\varphi \in J^\star$ s.t. $\sum_{J \in \mathbf{J}} H(J^\star, J) \leqslant K$?	

Theorem

$\textsc{WinDet}^\star(\text{DBP})$ *is NP-complete.*

Proof.

- Membership: write an integer program that solves it
- Hardness: reduction from *KemenyScore* (Bartholdi et al. 1989)

Conjecture (hardness)

$\textsc{Manipulability}(\text{DBP}^t)$ *is* Σ_2^p*-complete.*

Conclusions

In this work we define two judgment aggregation procedures:

1. Premise-based procedure (PBP)
   First vote on premises and then draw conclusions.

2. Distance-based procedure (DBP)
   The outcome minimizes the sum of the Hamming distance to the individual judgment sets.

And we study the complexity of winner determination and manipulation:

	WinDet	Manipulability
Pbp	P	NP-complete
Dbp	NP-complete	Σ_2^p-complete??