Group-Strategyproof Irresolute Social Choice Functions

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PREFERENCE AGGREGATION IN MULTIAGENT SYSTEMS

Preliminaries

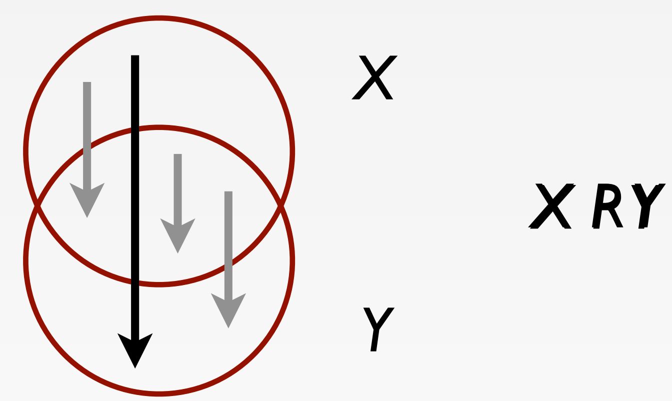
- Finite set of at least three alternatives
  - Each voter has complete preference relation R over alternatives
  - P: asymmetric part of R, I: symmetric part of R
- A social choice function (SCF) is a function that maps a preference profile to a non-empty subset of alternatives.
  - An SCF f is resolute if $|f(R)|=1$ for all preference profiles R.
  - A Condorcet extension is an SCF that uniquely chooses the Condorcet winner whenever one exists.
- An SCF is strategyproof (or non-manipulable) if no voter can obtain a more preferred outcome by misrepresenting his preferences.
  - An SCF is group-strategyproof if no group of voters can obtain an outcome that all of them prefer to the original one.

There cannot be only one

- Theorem (Gibbard, Satterthwaite; 1973, 1975): Every non-imposed, non-dictatorial, resolute SCF is manipulable.
- "*The Gibbard-Satterthwaite theorem on the impossibility of nondictorial, strategy-proof social choice uses an assumption of singlevaluedness which is unreasonable*" (Kelly; 1977)
- "*[resoluteness] is a rather restrictive and unnatural assumption*" (Gärdenfors; 1976)
- Problem: Resolute SCFs have to pick single alternatives based on the individual preferences only
  - incompatible with anonymity and neutrality

Lotteries and sets

- Gibbard (1977) characterized all strategyproof probabilistic SCFs
  - Winning alternative is chosen using a lottery with known probabilities
  - Voters have vNM preferences (utilities)
- Weakest model: Nothing is known about tie-breaking mechanism
  - $X\ R\ Y \Leftrightarrow \forall x \in X, y \in Y: (x\ R\ y)$ (Kelly; 1977)
    - $X\ P\ Y \Leftrightarrow \forall x \in X, y \in Y: (x\ R\ y) \wedge \exists x \in X, y \in Y: (x\ P\ y)$
  - Preference relation on sets is incomplete
  - $X\ R\ Y \Rightarrow \forall x,y \in X \cap Y: (x\ I\ y)$
  - Example: $a\ P\ b\ P\ c \Rightarrow \{a\}\ P\ \{a,b\}\ P\ \{b\}$
    - *$\{a,c\}$ and $\{b\}$ are incomparable*



- Many alternative (stronger) “preference extensions”
  - Fishburn (1972), Gärdenfors (1976), Pattanaik (1973), etc.

Yet another impossibility

- Theorem (Barbera, 1977; Kelly, 1977): Every non-imposed, non-dictatorial, quasi-transitively rationalizable SCF is manipulable.
- However, quasi-transitive rationalizability itself is highly problematic.
  - e.g., Gibbard (1969), Schwartz (1972), Mas-Colell/Sonnenschein (1972)
  - "*one plausible interpretation of such a theorem is that, rather than demonstrating the impossibility of reasonable strategy-proof social choice functions, it is part of a critique of the regularity [rationalizability] conditions*" (Kelly; 1977)
  - "*whether a nonrationalizable collective choice rule exists which is not manipulable and always leads to nonempty choices for nonempty finite issues is an open question*" (Barbera; 1977)

Results

- Every Condorcet extension is manipulable.
  - Strengthening of results by Gärdenfors (1976) and Taylor (2005)
- Every SCF that satisfies set-monotonicity and set-independence is weakly group-strategyproof.
- Every weakly strategyproof, pairwise SCF satisfies set-monotonicity and set-independence.

A pairwise SCF is weakly group-strategyproof iff it satisfies set-monotonicity and set-independence.

Every Condorcet extension is manipulable

R

2	2	2	1	1	1
bc	ac	ab	b	c	a
			c	a	b
a	b	c	a	b	c

wlog: $b \in f(R)$

R'

2	1	1	2	1	1	1
bc	a	ac	ab	b	c	a
	c			c	a	b
a	b	b	c	a	b	c

Case 1: $b \notin f(R') \Rightarrow$ Red voter manipulates (R ➠ R')

Case 2: $b \in f(R')$

R''

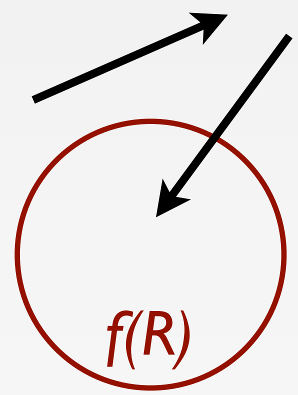
2	1	1	2	1	1	1
bc	a	a	ab	b	c	a
	c	c		c	a	b
a	b	b	c	a	b	c

Condorcet: $\{a\}=f(R'') \Rightarrow b \notin f(R)$

$\Rightarrow$ Blue voter manipulates (R' ➠ R'')

A characterization

- Previous example relied on breaking ties strategically.
  - An SCF is weakly group-strategyproof if no group can manipulate by only misrepresenting their strict preferences.
- Two new axioms
  - An SCF satisfies set-independence if modifying preferences between unchosen alternatives has no effect.
  - An SCF satisfies set-monotonicity if strengthening a chosen alternative against an unchosen one has no effect.



- Theorem: Every SCF that satisfies set-monotonicity and set-independence is weakly group-strategyproof.
  - Proof sketch: Induction over pairs of alternatives with misrepresented preferences, case analysis.

Consequences

group-strategyproof	*manipulable*
Pareto rule	essentially everything else
Omninomination rule	
Top cycle	
Minimal covering set (MC)	
Bipartisan set (BP)	
Tournament equilibrium set (TEQ) [subject to 20-year old conjecture]	

Pairwise SCFs

- An SCF is pairwise if it only depends on the difference of the number of voters who prefer a to b and those who prefer b to a for every pair of alternatives a and b (Young; 1974)
  - Examples
    - *Kemeny's rule, Borda's rule, Maximin, ranked pairs, all tournament solutions (Slater set, uncovered set, Banks set, minimal covering set, bipartisan set, TEQ, etc.)*
- Theorem: Every weakly strategyproof, pairwise SCF satisfies set-monotonicity and set-independence.
  - Proof sketch: Take preference profile that shows a failure of set-monotonicity or set-independence and construct a preference profile with two additional voters where one voter can manipulate.

Summary: A case for MC and BP

- Resistance to Manipulation
  - Strategic manipulation
    - misrepresenting preferences (resistance: *SP*)
    - abstaining election (resistance: *PA*)
  - Agenda manipulation
    - adding/deleting losing alternatives (resistance: *SSP*)
    - adding clones (strong resistance: *CC*)

	Kelly's extension			
	SP	*PA*	*SSP*	*CC*
Plurality	-	✓	-	-
Borda	-	✓	-	-
Copeland	-	-	-	-
MC	✓	✓	✓	✓
BP	✓	✓	✓	✓

- MC and BP have been axiomatized using SSP and CC.
- Computational aspects
  - MC and BP can be computed efficiently.
  - Is it possible to devise random selection protocols that prohibit meaningful prior distributions?