Consensus Measures Generated by Weighted Kemeny Distances on Linear Orders

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Extended Abstract

In the field of Social Choice, Bosch [4] introduced the notion of *consensus measure* as a mapping that assigns a number between 0 and 1 to every profile of linear orders, satisfying three properties: *unanimity* (in every subgroup of agents, the highest degree of consensus is only reached whenever all individuals have the same ranking), *anonymity* (the degree of consensus is not affected by any permutation of agents) and *neutrality* (the degree of consensus is not affected by any permutation of alternatives).

In García-Lapresta and Pérez-Román [8] we extended Bosch's notion of consensus measure to the context of weak orders (indifference among different alternatives is allowed) and we consider some additional properties that such measures could fulfill: *maximum dissension* (in each subset of two agents, the minimum consensus is only reached whenever preferences of agents are linear orders and each one is the inverse of the other), and *reciprocity* (if all individual weak orders are reversed, then the consensus does not change). After that, a class of consensus measures based on the distances among individual weak orders were introduced and analyzed. See also García-Lapresta and Pérez-Román [7].

In this contribution, we consider the above mentioned framework and properties for the case of linear orders. However, we now deal with the possibility of weighting discrepancies among linear orders by taking into account where these discrepancies appear. Since in some decision problems it is not the same to have differences in the top alternatives than in the bottom ones (see Baldiga and Green [3]), we introduce weights for distinguishing where these differences occur. To do this, we consider a class of consensus measures generated by weighted Kemeny distances, and we analyze some of their properties. The Kemeny metric was initially defined on linear orders by Kemeny [9], as the number of pairs where the orders' preferences disagree. We note that the Kemeny distance is a metric, but the introduced weighted Kemeny distances are not metrics in the sense of Deza and Deza [5]. On the the use of Kemeny and other metrics in the field of Social Choice see Eckert and Klamler [6].

Recently, Alcalde-Unzu and Vorsatz [1, 2] have introduced some consensus measures in the context of linear orders –related to some rank correlation indices– and they provide some axiomatic characterizations. It is important to note that both papers introduce a preliminary analysis to the weighting approach of consensus measures in the context of linear orders. See also Baldiga and Green [3].

It is interesting to note that the introduced consensus measures generated by weighted Kemeny distances can be used for designing appropriate decision making processes that require a minimum agreement among agents. For instance, in García-Lapresta and Pérez-Román [7] we propose a voting system where agents' opinions are weighted by the marginal contributions to consensus.

With respect to the computational aspect, we are preparing a computer program to obtain the consensus in real decisions when agents rank order the feasible alternatives. We are also working in an extension of the weighted consensus measures to the framework of weak orders.

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