Where Are the Hard Manipulation Problems?

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Abstract

One possible escape from the Gibbard-Satterthwaite theorem is computational complexity. For example, it is NP-hard to compute if the STV rule can be manipulated. However, there is increasing concern that such results may not reflect the difficulty of manipulation in practice. In this tutorial, I survey recent results in this area.

The Gibbard Satterthwaite theorem proves that, under some simple assumptions, a voting rule can always be manipulated. A number of possible escapes have been suggested. For example, if we relax the assumption of an universal domain and replace it with single peaked preferences, then strategy free voting rules exist. In an influential paper [1], Bartholdi, Tovey and Trick proposed that complexity might offer another escape: perhaps it is computationally so difficult to find a successful manipulation that agents have little option but to report their true preferences? Many voting rules have subsequently been shown to be NP-hard to manipulate [3]. However, NP-hardness only dictates the worst-case and may not reflect the difficulty of manipulation in practice. Indeed, a number of recent theoretical results suggest that manipulation can often be easy (e.g. [19]).

I argue here that we can study the hardness of manipulation empirically [17, 18]. There are several reasons why empirical analysis is useful. For example, theoretical analysis is usually restricted to simple distributions like uniform votes. Votes in real elections may be very different due, for instance, to correlations between votes. As a second example, theoretical analysis is often asymptotic so does not reveal the size of hidden constants. Such constants may be important to the actual computational cost. In addition, elections are typically bounded in size so asymptotic results may be uninformative. Such experiments suggest different behaviour occurs in the problem of computing manipulations of voting rules than in other NP-hard problems like propositional satisfiability [2, 13], constraint satisfaction [4, 9], number partitioning [6, 8], and other NP-hard problems [7, 14, 15]. For instance, many transitions seen in our experiments appear smooth, as seen in polynomial problems [16].

Another problem in which manipulation may be an issue is the stable marriage problem. Can agents be married to a more preferred partner by mis-reporting their preferences? Unfortunately, Roth [11] proved that *all* stable marriage procedures can be manipulated. We might hope that computational complexity might also be a barrier to manipulate stable marriage procedures. In joint work with Pini, Rossi and Venable, I have proposed a new stable marriage procedures based on voting that is NP-hard to manipulate [10]. This procedure has other desirable properties like gender neutrality.

A third domain in which manipulation may be an issue is sporting tournaments [12]. Manipulating a sporting tournament is slightly different to manipulating an election. In a sporting tournament, the voters are also the candidates. Since it is hard (without bribery or similar mechanisms) for a team to play better than it can, we consider just manipulations where the manipulators can throw games. We show, for example, that we can decide how to manipulate round robin and cup competitions, two of the most popular sporting competitions in polynomial time.

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