Allocation via Deferred-Acceptance under Responsive Priorities

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This extended abstract summarizes Ehlers and Klaus (2009)

1 Extended Abstract

We study the allocation of indivisible objects with capacity constraints to a set of agents when each agent receives at most one object and monetary compensations are not possible. Important applications of this model are the assignment of students to public schools, university admissions, and university housing allocation. We assume that students in these situations have strict preferences over the (object) types (e.g., admission to a specific school or university or dormitory rooms of a certain type) and that (object) types might come with a capacity constraint (the maximal number of students a school or university can admit or the maximal number of dormitory rooms of the same type). An allocation rule is a systematic way of solving any allocation problem (with capacity constraints).

In most papers that study the allocation of indivisible objects with capacity constraints, externally prescribed priorities are also specified; this class of problems is usually referred to as "school choice problems" or "student placement problems". Balinski and Sönmez (1999) were the first to formulate the allocation problem based on priorities, which in many real life situation naturally arise, e.g., in school choice students who live closer to a school and/or have siblings attending a school have higher priority at that school. The agents' priorities for a certain type are captured by an ordering of the agents: a priority structure. Given agents' priorities, it is natural to require that the allocation is "stable" with respect to the priorities. This means that there should be no agent who—conditional on higher priority—envies another agent (for receiving a better object). Given a priority structure, Gale and Shapley's (1962) famous deferred acceptance algorithm (an algorithm which has been extensively applied in practice, see Roth, 2008) can be used to find the agent-optimal stable allocation for any problem with capacity constraints and responsive priorities. We call a rule which is based on the agents-proposing deferred-acceptance algorithm with responsive priorities a *responsive DA-rule*.

Note that we do not a priori assume that priorities are externally given. Two other papers that consider this more general model of object allocation with multiple copies of each type and capacity constraints are Ehlers and Klaus (2006) and Kojima and Manea (2009). Kojima and Manea (2009) point out that "Despite the importance of deferred acceptance rules in both theory and practice, no axiomatization has yet been obtained in an object allocation setting with unspecified priorities." Then, they proceed to provide two characterizations of deferred acceptance rules with so-called acceptant substitutable priorities (a larger class of rules than the class of responsive DA-rules which is based on priorities that are determined by a choice function that reflects substitutability in preferences over sets of agents).

We consider situations where resources may change, i.e., it could be that additional objects are available. When the change of the environment is exogenous, it would be unfair if the agents who were not responsible for this change were treated unequally. We apply this idea of solidarity and require that if additional resources become available, then all agents (weakly) gain. This requirement is called *resource-monotonicity*. Next, we add the mild efficiency requirement of *weak non-wastefulness* as well as the very basic and intuitive properties of *individual rationality* and *unavailable type invariance*. We also impose the invariance property *truncation invariance*. Our last property is the well-known strategic robustness condition of *strategy-proofness*. First, we show that these elementary and intuitive properties characterize, for so-called house allocation problems (quotas at most one), the class of responsive DA-rules that are based on the agent-proposing deferred-acceptance algorithm with responsive priority structures (Theorem 1). Second, we extend this characterization to the class of all problems with capacity constraints, by replacing *resource-monotonicity* with the new property of *two-agent consistent conflict resolution* (Theorem 2).

Another situation of interest is the change of the set of agents and objects because agents leave with their allotments. *Consistency* requires that the allocation for the "reduced economy" allocates the remaining objects to the remaining agents in the same way as before. Since many rules do not satisfy *consistency*, we introduce *weak consistency*, which only requires that agents who received the null object in the original economy still receive the null object in any reduced economy. We obtain a third characterization of the class of responsive DA-rules by *unassigned type invariance*, *individual rationality*, *weak non-wastefulness*, *weak consistency*, and *strategy-proofness* (Theorem 3).

References

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