Collective Attention and Ranking Methods

Extended Abstract

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The use of rankings is becoming pervasive in many areas including academia for ranking researchers, journals, universities, and the Web environment for ranking Internet pages. The public good aspect of information explains the use of rankings. Rankings are based on a costly process of gathering and summarizing some relevant information on the alternatives in a particular topic. When such information is relevant to anyone, the publication of rankings avoids each individual to pay the search and processing costs. For that very reason, rankings have some influence on the attention that is devoted to the various alternatives. In recurrent situations, attention will, in turn, alter the new statements on which subsequent rankings will be based. This paper proposes an analysis of the feedback between rankings, attention intensities, and statements by studying some reasonable dynamics.

A ranking problem is described by a set of items to be ranked and a set of 'experts' who provide some statements on which the ranking will be based. Rankings here are cardinal, meaning that relative scores are assigned to items. In some situations, as in the ranking of Web pages based on the link structure, the items to be ranked coincide with the experts. These situations are sometimes referred to as the judgment by 'peers'.

The analysis bears on ranking methods that satisfy two important properties. The first property, *intensity invariance*, has been introduced for dealing with the situations in which the 'intensity' of statements is not controlled. In such situations, one may not want an expert to increase its impact on the final ranking by an inflation in its statements (there are other justifications, as explained in the paper). An 'intensity invariant' ranking method is obtained by factoring out the intensity of experts' statements. For example, the 'invariant' method, which serves as a basis to PageRank of Google, factors out the intensity of outward links to avoid pages to increase their score by inflating the number of these links.

The second property, that of *supporting weights* views a method as simultaneously assigning scores to the items and weights to the experts. Given the experts' statements, the ranking writes as a weighted combination of the experts' statements in which furthermore the scores and the weights form some sort of an equilibrium relationship. The property is satisfied by most current methods - e.g. the counting method, the invariant method, the Hits method- although it has not be made explicit so far. This property is useful for various reasons. In particular, it helps us to define new methods through alternative equilibrium relationships and to give a precise definition to what a peers' method is.

The first part of the paper considers static problems, in which the experts' statements are given. I introduce a new ranking method that is both intensity invariant and supported by equilibrium weights. The equilibrium is based on the notion of handicaps. There are indeed strong relationships between rankings and handicaps. Since the purpose of handicaps is to adjust the marks received by items so as to equalize their 'strength', rankings and handicaps are inversely related to each other. The method, called the *handicap-based method*, is characterized by simple properties. The computation of the handicap-based ranking relies on a well-known procedure of matrix scaling, called RAS method or iterative proportional fitting procedure.

The second part of the paper studies a recurrent framework to analyze the influence of rankings. This influence is driven by their impact on attention intensities. In a context in which the number of alternatives to consider is huge, experts cannot carefully assess each one and tend to pay more attention to those whose score is higher. For example, while working

on a paper, a researcher who uses rankings tends to read more the journals whose ranks are higher. An 'influence function' describes how the current ranking modifies attention intensities. This generates a joint dynamics on rankings and statements because statements depend on both preferences and attention: the current ranking modifies attention intensities, hence the next statements on which next ranking is based. An intuition is that, as past statements have an impact on future statements through rankings computation, we might expect 'the rich to get richer'. However, the impact of such self-enforcing mechanism may differ according to the ranking method. Our aim is to investigate more precisely this link between a ranking method and the dynamics, starting with a simple linear form for the influence function. Contrasted results are obtained for two different classes of methods.

The first class, called the generalized handicap-based methods, is obtained from the handicap-based method by modifying the experts' weights. The class includes both the handicap-based and the counting methods. These methods guarantee stability in the sense that, given preferences for the experts, the sequence of rankings converges towards a unique rest point.

The second class is the class of peers' methods. The rationale behind a peers' method is that the ability of an individual to perform (measured by his score) is correlated with his ability to judge others' performance. In particular, for a method supported by weights, a minimal requirement is that an individual who receives a small score is also assigned a small expert's weight. This defines a peers' method. I show that whatever peers' method, the dynamics may admit multiple limit points for some preferences, each one corresponding to a different support (the support is the subset of items that keep a positive score). Furthermore, the supports of the limit points are independent of the peers' method. Such result illustrates the self-sustaining aspect of a peers' method. Self-sustainability here is not obtained through plain manipulation but through the coordination device induced by the influence of the ranking.[1]

This paper is about the convergence of behaviors and statements. This is also the concern of the large literature that analyzes the influence of opinions channelled by 'neighbors' in a partially connected network. This literature analyzes situations in which individuals receive private signals about a state of the world. One main question is whether (non-strategic) communication will lead opinions to converge to a common belief and, if convergence occurs, how this common belief relates to the initial opinions and the network structure. Instead here information -the ranking- is made public and influences all experts in an identical way. The impact however differs across experts because they differ in their preferences.The analysis shows that the interplay of preferences and the ranking method may induce a variety of different outcomes.

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[1]Researchers in computer science have also concerns about the influence of the rankings provided by search engines. The main criticism is that rankings are biased towards already popular webpages, thus preventing the rise in popularity of recently created high quality pages. There has been some proposals to correct the bias, such as introducing some randomness in the rankings, or to account of the date of creation of a page in the computation of the ranking.