On the Fixed-Parameter Tractability of Composition-Consistent Tournament Solutions

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Abstract

Tournament solutions, i.e., functions that associate with each complete and asymmetric relation on a set of alternatives a non-empty subset of the alternatives, play an important role within social choice theory and the mathematical social sciences at large. Laffond et al. have shown that various tournament solutions satisfy *composition-consistency*, a strong structural invariance property based on the similarity of alternatives. We define the *decomposition degree* of a tournament as a parameter that reflects its decomposability and show that computing any composition-consistent tournament solution is fixed-parameter tractable with respect to the decomposition degree. This is of particular relevance for tournament solutions that are known to be computationally intractable such as the Banks set and the tournament equilibrium set, both of which have been proposed in the context of social choice. Finally, we experimentally investigate the decomposition degree of two natural distributions of tournaments.

1 Introduction

Many problems in multiagent decision making can be addressed using tournament solutions, i.e., functions that associate with each complete and asymmetric relation on a set of alternatives a non-empty subset of the alternatives. Tournament solutions are most prevalent in social choice theory, where the binary relation is typically assumed to be given by the simple majority rule (Moulin, 1986; Laslier, 1997). Other application areas include multi-criteria decision analysis (Arrow and Raynaud, 1986; Bouyssou et al., 2006), zero-sum games (Fisher and Ryan, 1995; Laffond et al., 1993; Duggan and Le Breton, 1996), coalition formation (Brandt and Harrenstein, 2011), and argumentation theory (Dung, 1995; Dunne, 2007).

Recent years have witnessed an increasing interest in the computational complexity of tournament solutions by the multiagent systems and theoretical computer science communities. A number of concepts such as the Banks set (Woeginger, 2003), the Slater set (Alon, 2006; Conitzer, 2006), and the tournament equilibrium set (Brandt et al., 2010) have been shown to be computationally intractable. For others, including the minimal covering set and the bipartisan set, algorithms that run in polynomial time but are nevertheless computationally quite demanding because they rely on linear programming, have been provided (Brandt and Fischer, 2008). The class of all tournaments is excessively rich and it is well-known that only a fraction of these tournaments occur in realistic settings (see, e.g., Feld and Grofman, 1992). Therefore, an important question is whether there are natural classes or distributions of tournaments that admit more efficient algorithms for computing specific tournament solutions. In this paper, we study tournaments that are *decomposable* in a certain well-defined way. A set of alternatives forms a component if all alternatives in this set bear the same relationship to all outside alternatives. Elements of a component can thus be seen as variants of the same type of an alternative. Laslier (1997) has shown that every tournament admits a unique natural decomposition into components, which may themselves be decomposable into subcomponents. A tournament solution is *composition-consistent* if it chooses the best alternatives of the best components (Laffond et al., 1996).[1] In other words, a composition-consistent tournament solution can be computed by recursively determining the winning components. All of the tournament solutions mentioned earlier except the Slater set are composition-consistent.

In this paper, we provide a precise formalization of the recursive decomposition of tournaments and a detailed analysis of the speed-up that can be achieved when computing composition-consistent tournament solutions. In particular, we define the *decomposition degree* of a tournament as a parameter that reflects its decomposability. Intuitively, a low decomposition degree indicates that the tournament admits a particularly well-behaved decomposition and therefore allows the efficient computation of composition-consistent tournament solutions. Within our analysis, we leverage a recently proposed linear-time algorithm for the modular decomposition of directed graphs (McConnell and de Montgolfier, 2005; Capelle et al., 2002).

In related work, Betzler et al. (2010) proposed data reduction rules that facilitate the computation of *Kemeny rankings*. One of these rules, the "Condorcet-set rule", corresponds to a (rather limited) special case of composition-consistency where tournaments are decomposed into exactly *two* components. Furthermore, a preprocessing technique that resembles the one proposed in this paper has been used by Conitzer (2006) to speed up the computation of *Slater rankings*. Interestingly, even though Slater's solution is *not* composition-consistent, decompositions of the tournament can be exploited to identify a *subset* of the optimal rankings.

Our results, on the other hand, allow us to compute *complete choice sets* and are applicable to *all composition-consistent tournament solutions*, including the uncovered set (Fishburn, 1977; Miller, 1980), the minimal covering set (Dutta, 1988), the bipartisan set (Laffond et al., 1993), the Banks set (Banks, 1985), the tournament equilibrium set (Schwartz, 1990), and the minimal extending set (Brandt, 2009). The former three admit polynomial-time algorithms whereas the latter three are computationally intractable. None of the concepts is known to admit a linear-time algorithm.

We show that computing any composition-consistent tournament solution is *fixed-parameter tractable* with respect to the decomposition degree of the tournament, i.e., there are algorithms that are only superpolynomial in the decomposition degree. We conclude the paper with an extensive investigation of the decomposition degree of two natural distributions of tournaments. The first one is a well-studied model model that assumes the existence of a true linear ordering of the alternatives that has been perturbed by binary random inversions. The other one is a spatial voting model based on the proximity of voters and alternatives in a multi-dimensional space.

2 Preliminaries

In this section, we provide the terminology and notation required for our results (see Laslier (1997) for an excellent overview of tournament solutions and their properties).

2.1 Tournaments

Let X be a universe of alternatives. For notational convenience we assume that $\mathbb{N} \subseteq X$. The set of all non-empty finite subsets of X will be denoted by $\mathcal{F}(X)$. A (finite) *tournament* T is a pair $(A, \succ)$, where $A \in \mathcal{F}(X)$ and $\succ$ is an asymmetric and complete (and thus irreflexive)

[1] Composition-consistency is related to *cloning-consistency*, which was introduced by Tideman (1987) in the context of social choice.

binary relation on X, usually referred to as the *dominance relation*.[2] Intuitively, $a \succ b$ signifies that alternative a is preferable to b. The dominance relation can be extended to sets of alternatives by writing $A \succ B$ when $a \succ b$ for all $a \in A$ and $b \in B$.[3] We further write $\mathcal{T}(X)$ for the set of all tournaments on X. The *order* $|T|$ of a tournament $T = (A, \succ)$ refers to its number of alternatives $|A|$. Finally, a *tournament isomorphism* of two tournaments $T = (A, \succ)$ and $T' = (A', \succ')$ is a bijective mapping $\pi : A \to A'$ such that $a \succ b$ if and only if $\pi(a) \succ' \pi(b)$.

2.2 Components and Decompositions

An important structural concept in the context of tournaments is that of a *component*. A component is a subset of alternatives that bear the same relationship to all alternatives not in the set.

Definition 1. Let $T = (A, \succ)$ be a tournament. A non-empty subset B of A is a *component* of T if for all $a \in A \setminus B$ either $B \succ a$ or $a \succ B$. A *decomposition* of T is a set of pairwise disjoint components $\{B_1, \dots, B_k\}$ of T such that $A = \bigcup_{i=1}^{k} B_i$.

The *null decomposition* of a tournament $T = (A, \succ)$ is $\{A\}$; the *trivial decomposition* consists of all singletons of A. Any other decomposition is called *proper*. A tournament is said to be *decomposable* if it admits a proper decomposition. Given a particular decomposition, the *summary* of a tournament is defined as the tournament on the individual components rather than the alternatives.

Definition 2. Let $T = (A, \succ)$ be a tournament and $\tilde{B} = \{B_1, \dots, B_k\}$ a decomposition of T. The *summary* of T with respect to $\tilde{B}$ is defined as $\tilde{T} = (\{1, \dots, k\}, \tilde{\succ})$, where

$$i \mathrel{\tilde{\succ}} j \quad \text{if and only if} \quad B_i \succ B_j.$$

A tournament is called *reducible* if it admits a decomposition into *two* components. Otherwise, it is *irreducible*. Laslier (1997) has shown that there exist a natural unique way to decompose any tournament. Call a decomposition $\tilde{B}$ *finer* than another decomposition $\tilde{B}'$ if $\tilde{B} \neq \tilde{B}'$ and for each $B \in \tilde{B}$ there exists $B' \in \tilde{B}'$ such that $B \subseteq B'$. $\tilde{B}'$ is said to be *coarser* than $\tilde{B}$. A decomposition is *minimal* if its only coarser decomposition is the null decomposition.

Proposition 1 (Laslier (1997)). *Every irreducible tournament with more than one alternative admits a unique minimal decomposition.*

This is obviously not true for *reducible* tournaments, as witnessed by the tournament $T = (\{1, 2, 3\}, \succ)$ with $1 \succ 2$, $1 \succ 3$, and $2 \succ 3$, which admits two minimal decompositions, namely $\{\{1\}, \{2, 3\}\}$ and $\{\{1, 2\}, \{3\}\}$. Nevertheless, there is a unique way to decompose any reducible tournament. A *scaling decomposition* is a decomposition with a transitive summary.

Proposition 2 (Laslier (1997)). *Every reducible tournament admits a unique scaling decomposition such that each component is irreducible.*

This scaling decomposition into irreducible components is also the finest scaling decomposition.

[2] This definition slightly diverges from the common graph-theoretic definition where $\succ$ is defined on A rather than X. However, it facilitates the sound definition of tournament solutions.

[3] To avoid cluttered notation, we omit the curly braces if one of the sets is a singleton, i.e., we write $a \succ B$ instead of the more cumbersome $\{a\} \succ B$.

2.3 Tournament Solutions

A *maximal* element of a tournament $T = (A, \succ)$ is an alternative that is not dominated by any other alternative. Due to the asymmetry of the dominance relation, there can be at most one maximal element, which then also constitutes a *maximum*. Let $\max(T)$ denote the function that yields the empty set or the maximum whenever one exists, i.e.,

$$\max(T) = \{a \in A : a \succ b \text{ for all } b \in A \setminus \{a\}\}.$$

In social choice theory, the maximum of a tournament given by a majority relation is commonly referred to as the *Condorcet winner*.

Since the dominance relation may contain cycles and thus fail to have a maximal element, a variety of concepts have been suggested to take over the role of singling out the "best" alternatives of a tournament. Formally, a *tournament solution* S is defined as a function that associates with each tournament $T = (A, \succ)$ a non-empty subset $S(T)$ of A. Following Laslier (1997), we require a tournament solution to be independent of alternatives outside the tournament, invariant under tournament isomorphisms, and to select the maximum whenever it exists.

Definition 3. A *tournament solution* is a function $S : \mathcal{T}(X) \to \mathcal{F}(X)$ such that

(*i*) $S(T) \subseteq A$ for all tournaments $T = (A, \succ)$;

(*ii*) $S(T) = S(T')$ for all tournaments $T = (A, \succ)$ and $T' = (A, \succ')$ such that $T|_A = T'|_A$;

(*iii*) $S((\pi(A), \succ')) = \pi(S((A, \succ)))$ for all tournaments $(A, \succ)$, $(A', \succ')$, and every tournament isomorphism $\pi : A \to A'$ of $(A, \succ)$ and $(A', \succ')$; and

(*iv*) $S(T) = \max(T)$ whenever $\max(T) \neq \emptyset$.

A tournament solution is composition-consistent if it chooses the "best" alternatives from the "best" components (Laffond et al., 1996).

Definition 4. A tournament solution S is *composition-consistent* if for all tournaments T and $\tilde{T}$ such that $\tilde{T}$ is the summary of T with respect to some decomposition $\{B_1, \ldots, B_k\}$,

$$S(T) = \bigcup_{i \in S(\tilde{T})} S(T|_{B_i}).$$

2.4 Fixed-Parameter Tractability and Parameterized Complexity

We briefly introduce the most basic concepts of parameterized complexity theory (see, e.g., Downey and Fellows, 1999; Niedermeier, 2006). In contrast to classical complexity theory, where the size of problem instances is the only measure of importance, parameterized complexity analyzes whether the hardness of a problems only depends on the size of certain parameters. A problem with parameter k is said to be *fixed-parameter tractable* (or to belong to the class FPT) if there exists an algorithm that solves the problem in time $f(k) \cdot poly(|I|)$, where $|I|$ is the size of the input and f is some computable function independent of $|I|$.

For example, each (computable) problem is trivially fixed-parameter tractable with respect to the parameter $|I|$. The crucial point is to identify a parameter that is reasonably small in realistic instances and to devise an algorithm that is only superpolynomial in this parameter.

3 The Decomposition Tree of a Tournament

Propositions 1 and 2 offer a straightforward method to iteratively decompose tournaments. If the tournament is reducible, take the scaling decomposition with irreducible components. If it is irreducible, take the minimal decomposition. The repeated application of these decompositions leads to the *decomposition tree* of a tournament.

Definition 5. The *decomposition tree* $D(T)$ of a tournament $T = (A, \succ)$ is defined as a rooted tree whose nodes are non-empty subsets of A. The root of $D(T)$ is A and for each node $B \in C$ with $|B| \geq 2$, the children of B are defined as follows:

- If $T|_B$ is reducible, the children of B are the components of a finest scaling decomposition of $T|_B$.
- If $T|_B$ is irreducible, the children of B are the components of a minimal decomposition of $T|_B$.

It also follows from Propositions 1 and 2 that every tournament has a *unique* decomposition tree. By definition, each node in $D(T)$ is a component of T and each leaf is a singleton. However, not all components of T need to appear as nodes in $D(T)$. An example of a decomposition tree is provided in Figure 1.

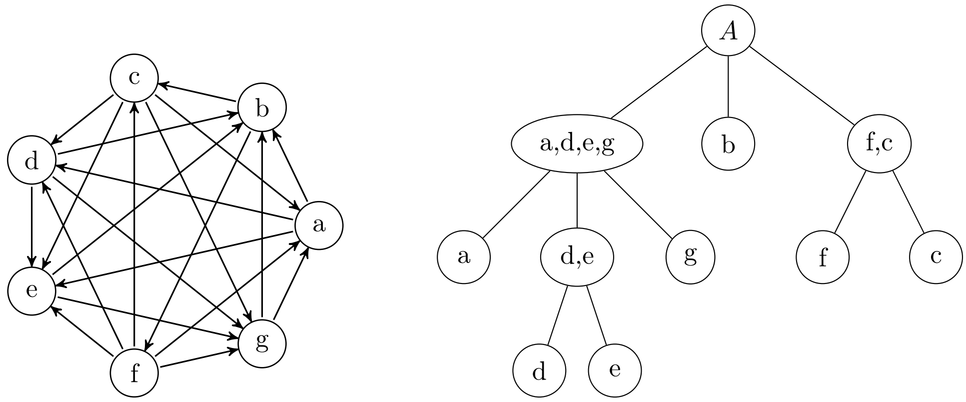

Figure 1: Example tournament with corresponding decomposition tree. Nodes $\{f, c\}$ and {d,e} are reducible, all other nodes are irreducible. Curly braces are omitted to improve readability.

An internal (i.e., non-leaf) node B of $D(T)$ with children $B_1, \ldots, B_k$ corresponds to the tournament $T_B = (\{1, \ldots, k\}, \tilde{\succ})$ where $i \mathrel{\tilde{\succ}} j$ if and only if $B_i \succ B_j$, i.e., T_B is the summary of $T|_B$ with respect to the decomposition $\{B_1, \ldots, B_k\}$. The order of T_B is thus equal to the number of children of node B. Moreover, we call an internal node B *reducible* (respectively, *irreducible*) if the tournament T_B is reducible (respectively, irreducible).[4] If B is reducible, we assume without loss of generality that the children $B_1, \ldots, B_k$ are labelled according to their transitive summary, i.e., $B_i \succ B_j$ if and only if $i < j$. In particular, $\max(T_B) = \{1\}$.

Recent results on the modular decomposition of directed graphs (Capelle et al., 2002; McConnell and de Montgolfier, 2005) imply that the decomposition tree of a tournament can be computed in linear time.[5]

Proposition 3. *The decomposition tree of a tournament T can be computed in time $O(|T|^2)$.*

[4] $T|_B$ is reducible (respectively, irreducible) if and only if its summary T_B is.

[5] The representation of a tournament is quadratic in the number of its alternatives.

The proof consists of two steps. In the first step, a *factorizing permutation* of the tournament is constructed. A factorizing permutation of $T = (A, \succ)$ is a permutation of the alternatives in A such that each component of T is a contiguous interval in the permutation. McConnell and de Montgolfier (2005) provide a simple algorithm that computes a factorizing permutation of a tournament in linear time. Furthermore, there exists a fairly complicated linear-time algorithm by Capelle et al. (2002) that, given a tournament T and a factorizing permutation of T, computes the decomposition tree $D(T)$. Since the literature on composition-consistency in social choice and on modular decompositions in graph theory is unfortunately not well-connected and for reasons of completeness, we outline both algorithms in the Appendix.

The concept of a factorizing permutation also yields a simple way to bound the number of nodes in the decomposition tree.

Lemma 1. *The number of* internal *nodes in the decomposition tree of a tournament T is at most $|T| - 1$.*

Proof. Let $\sigma(T)$ be a factorizing permutation of T and consider a node B in $D(T)$. Decomposing B into new components (the children of B in $D(T)$) corresponds to making "cuts" in $\sigma(T)$. Furthermore, each cut generates at most two new components.[6] As there are only $|T| - 1$ possible positions for such a cut, the maximum number of nodes in $D(T)$ is $1 + 2(|T| - 1) = 2|T| - 1$. The bound follows from the observation that $D(T)$ has exactly $|T|$ leaves. □

4 Computing Solutions via the Decomposition Tree

Let S be a composition-consistent tournament solution and consider an arbitrary tournament $T = (A, \succ)$ together with its decomposition tree $D(T)$. Composition-consistency implies that

$$S(T|_B) = \bigcup_{i \in S(T_B)} S(T|_{B_i}) \tag{1}$$

for each internal node B in $D(T)$ with children $B_1, \ldots, B_k$. The solution set $S(T)$ can thus be computed by starting at the root of $D(T)$ and iteratively applying equation 1. If B is reducible, we immediately know that $S(T|_B) = S(T|_{B_1})$, since 1 is the maximum in the transitive tournament T_B. A straightforward implementation of this approach is given in Algorithm 1.

Algorithm 1 visits each node of $D(T)$ at most once. The algorithm for computing S is only invoked for tournaments T_B for which B is irreducible. The order of such a tournament T_B is equal to the number of children of the node B in $D(T)$. The *decomposition degree* of T is defined as an upper bound of this number.

Definition 6. The *decomposition degree* $\delta(T)$ of a tournament T is given by

$$\delta(T) = max\{|T_B| : B \text{ is an irreducible internal node in } D(T)\}.$$

Proposition 3 implies that $\delta(T)$ can be computed efficiently. The decomposition degree of the example tournament in Figure 1 is 3.

Let $f(n)$ be an upper bound on the running time of an algorithm that computes $S(T)$ for tournaments of order $|T| \leq n$. Then, the running time of Algorithm 1 can be upper-bounded by $f(\delta(T))$ times the number of irreducible nodes of $D(T)$. We thus obtain the following theorem.

[6] Cuts can be made simultaneously, in which case the number of new components *per cut* is smaller.

Algorithm 1 Compute $S(T)$ via decomposition tree

Compute $D(T)$
$S, S' \leftarrow \emptyset$
$Q \leftarrow (A)$
while $Q \neq ()$ **do**
    $B \leftarrow$ Dequeue(Q)
    if $|B| = 1$ **then**
        $S \leftarrow S \cup B$
    else
        if B is reducible **then**
            Enqueue(Q, B_1)
        else // B is irreducible
            for all $i \in S(T_B)$ **do**
                Enqueue(Q, B_i)
return S

Theorem 1. *Let S be a composition-consistent tournament solution and let $f(k)$ be an upper bound on the running time of an algorithm that computes S for tournaments of order at most k. Then, $S(T)$ can be computed in $O(n^2) + f(\delta) \cdot (n-1)$ time, where δ is the decomposition degree of T and n is the order or T.*

Proof. Let T be a tournament and $n = |T|$. Computing $D(T)$ requires time $O(n^2)$ (Proposition 3). We now show that Algorithm 1 computes $S(T)$ in time $f(\delta(T)) \cdot (n-1)$. Correctness follows from composition-consistency of S. The running time can be bounded as follows. During the execution of the while-loop, each node B of $D(T)$ is visited at most once. If B is reducible or a singleton, there is no further computation. If B is irreducible, $S(T_B)$ is computed. As $|T_B|$ is upper-bounded by $\delta(T)$, this can be done in $f(\delta(T))$ time. Finally, Lemma 1 shows that the number of (internal) nodes of $D(T)$ is at most $n-1$. Summing up, this yields a running time of $O(n^2) + f(\delta(T)) \cdot (n-1)$. $\square$

In particular, Theorem 1 shows that the computation of $S(T)$ is fixed-parameter tractable with respect to the parameter $\delta(T)$.

To get a better understanding of this theorem, consider a composition-consistent tournament solution S such that $f(n)$ is in $\mathrm{E} = \mathrm{DTIME}(2^{O(n)})$. This holds, for example, for the Banks set. For given tournaments T of order n, Theorem 1 then implies that $S(T)$ can be computed efficiently (i.e., in time polynomial in n) whenever $\delta(T)$ is in $O(\log^k n)$. Theorem 1 is also applicable to tractable tournaments solutions such as the minimal covering set and the bipartisan set. Although computing these solutions is known to be in P, existing algorithms rely on linear programming and may be too time-consuming for very large tournaments. For both concepts, a significant speed-up can be expected for distributions of tournaments that admit a small decomposition degree.

Generally, decomposing a tournament asymptotically never harms the running time, as the time required for computing the decomposition tree is only linear in the input size.[7]

5 Experimental Results

It has been shown in the previous section that computing composition-consistent tournament solutions is fixed-parameter tractable with respect to the decomposition degree of a

[7] Checking whether there exists a maximum already requires $O(n^2)$ time.

tournament. While the clustering of alternatives within components has some natural appeal by itself, an important question concerns the value of the decomposition degree for reasonable and practically motivated distributions of tournaments. In this section, we will explore this question experimentally using two probabilistic models from social choice theory. Both models are based on a set of *voters* who entertain preferences over *candidates*. Given a finite set of candidates C and an odd number of voters with linear preferences over C, the *majority tournament* is defined as the tournament $(C, \succ)$, where $a \succ b$ if and only if the number of voters preferring a to b is greater than the number of voters preferring b to a.

Noise model The first model we consider is a standard model in social choice theory where it is usually attributed to Condorcet (see, e.g., Young, 1988). Condorcet assumed that there exists a "true" ranking of the candidates and that the voters possess noisy estimates of this ranking. In particular, he assumed that there is a probability $p > \frac{1}{2}$, such that for each pair a, b of candidates, each voter ranks a and b according to the true ranking with probability p and ranks them incorrectly with probability $1 - p$.

Spatial Model Spatial models of voting are well-studied objects in social choice theory (see, e.g., Austen-Smith and Banks, 2000). For a fixed natural number d of issues, we assume that candidates (i.e., alternatives) as well as voters are located in the space $[0, 1]^d$. The position of candidates and voters can be thought of as their stance on the d issues. Voters' preferences over candidates are given by the proximity to their own position according to the Euclidian distance. We generate tournaments by drawing the positions of candidates and voters uniformly at random from $[0, 1]^d$.

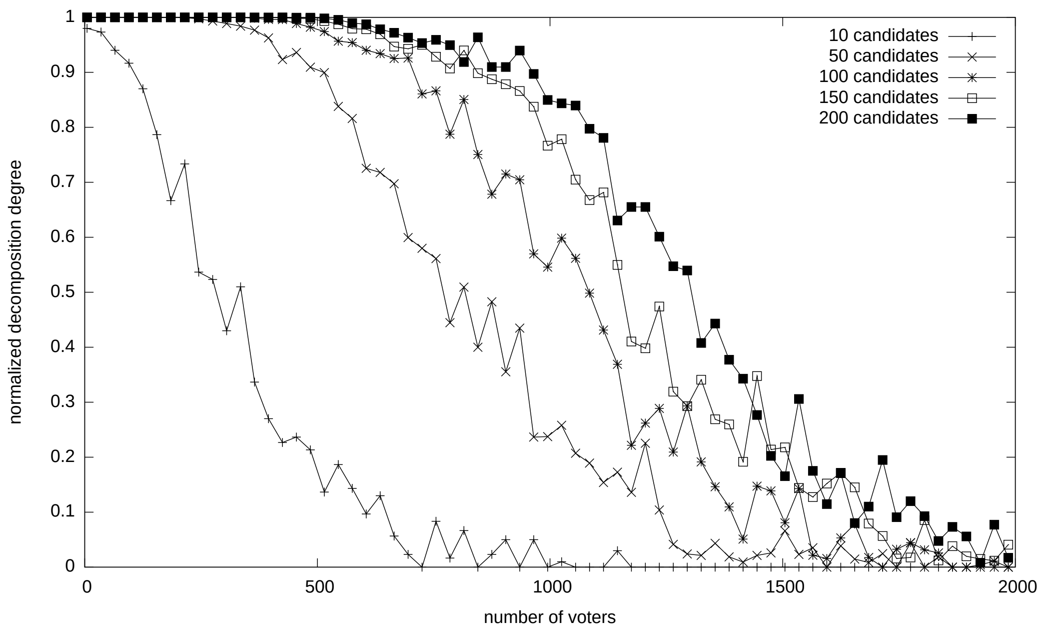

Figure 2: Noise model with $p = 0.55$

The results of our experiments are presented in Figures 2, 3, and 4. The x-axis shows the number of voters, which goes from 5 to 1985 in increments of 30. In order to facilitate the comparison of results for a varying number of candidates, the y-axis is labelled with the *normalized* decomposition degree, i.e., the decomposition degree divided by the number of candidates. Each graph shows the results for a fixed number of candidates, and each data point corresponds to the average value of 30 instances. Whenever the normalized decomposition degree is less than one, composition-consistency can be exploited, even for tournament solutions that already admit fast (say, linear-time) algorithms. The slower the original algorithm, the more dramatic is the speedup obtained by capitalizing on the decomposition tree.

Figure 2 shows the results for the noise model with parameter $p = 0.55$. For any number of candidates, the decomposition degree goes to zero when the number of voters grows. This is not surprising because the probability that the tournament is transitive tends to 1 for any $p > \frac{1}{2}$ (and a transitive tournament T has $\delta(T) = 0$). Interestingly, the decomposition degree drops abruptly when a certain number of voters is reached.

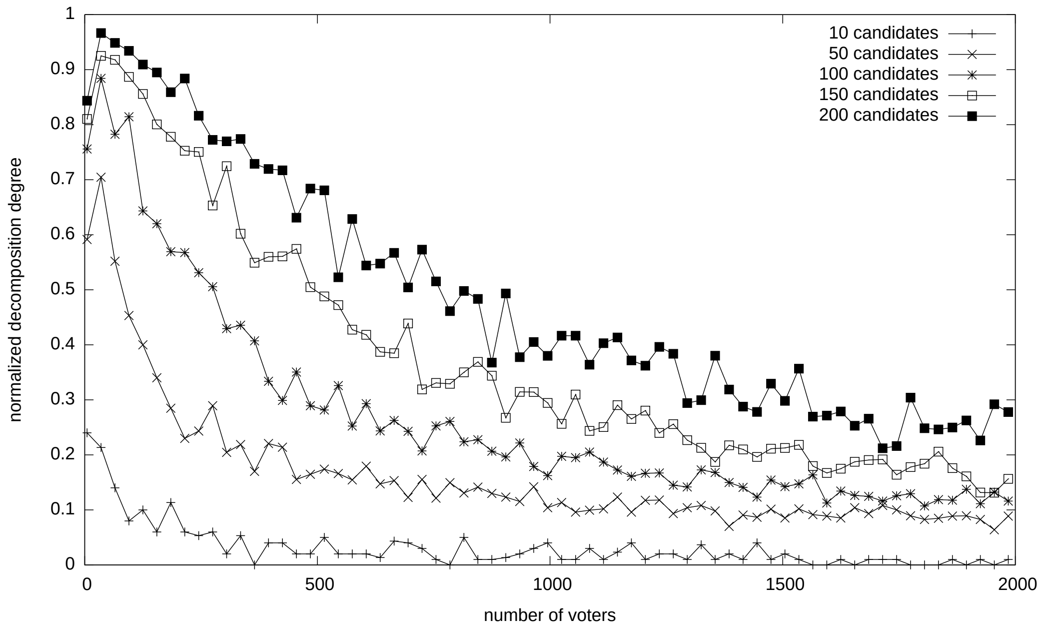

Figure 3: Spatial model with $d = 2$

Figures 3 and 4 show the results for the spatial model for dimensions $d = 2$ and $d = 20$. Surprisingly, the decomposition degree does not significantly increase when moving to a higher-dimensional space. Similar to the noise model discussed above, δ tends to 0 for growing n because a population of voters that is evenly distributed in $[0, 1]^d$ tends to produce transitive tournaments.

The results of our experiments show that, even for moderately-sized electorates, tournaments in both distributions are highly decomposable and therefore allow significantly faster algorithms for computing composition-consistent tournament solutions. For example, consider the two-dimensional spatial model with 150 candidates and some tournament solution that can be computed in time 2^n. For 500 voters, the (average) normalized decomposition degree is approximately 0.5. When assuming for simplicity that the decomposition tree is already given, the speed-up factor (i.e., the running time of the original algorithm divided by the running time of the algorithm that exploits composition-consistency) is $\frac{2^{150}}{2^{75} \cdot (150-1)} \approx 2.5 \cdot 10^{20}$.

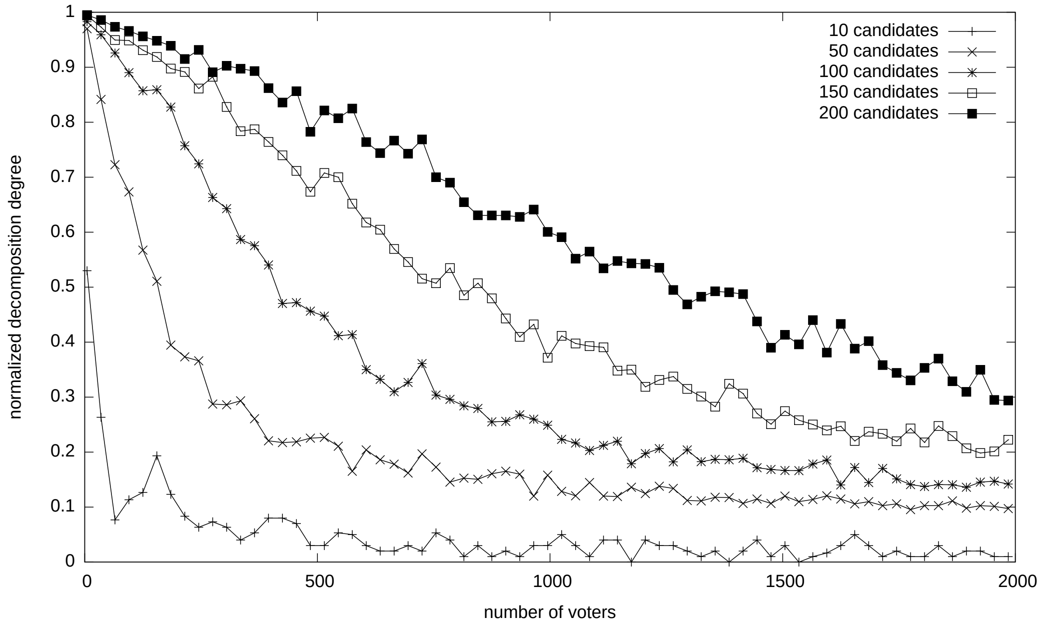

Figure 4: Spatial model with $d = 20$

6 Conclusion

In this paper, we studied the algorithmic benefits of composition-consistent tournament solutions. We defined the decomposition degree of a tournament as a parameter that reflects its decomposability. Intuitively, a low decomposition degree indicates that the tournament admits a particularly well-behaved decomposition. Our main result states that computing any composition-consistent tournament solution is fixed-parameter tractable with respect to the decomposition degree. This is of particular relevance for tournament solutions that are known to be computationally intractable such as the Banks set and the tournament equilibrium set. For example, one corollary of our main result is that the Banks set of a tournament can be computed efficiently whenever the decomposition degree is polylogarithmic in the number of alternatives. We experimentally determined the decomposition degree of two natural distributions of tournaments stemming from social choice theory and found that the decomposition degree in many realistic instances is surprisingly low. As a consequence, the speedup obtained by exploiting composition-consistency when computing tournament solutions for these instances will be quite substantial.

In future work, it would be interesting to measure the concrete effect of capitalizing on composition-consistency on the running time of existing algorithms for specific tournament solutions. Since computing a decomposition tree requires only linear time, it is to be expected that decomposing a tournament never hurts, and often helps. Composition-consistency can be further exploited by parallelization and storing the solutions of small tournaments in a lookup table.

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References

N. Alon. Ranking tournaments. *SIAM Journal on Discrete Mathematics*, 20(1):137–142, 2006.

K. J. Arrow and H. Raynaud. *Social Choice and Multicriterion Decision-Making*. MIT Press, 1986.

D. Austen-Smith and J. S. Banks. *Positive Political Theory I: Collective Preference*. University of Michigan Press, 2000.

J. S. Banks. Sophisticated voting outcomes and agenda control. *Social Choice and Welfare*, 3:295–306, 1985.

N. Betzler, R. Bredereck, and R. Niedermeier. Exact rank aggregation based on effective data reduction. Unpublished manuscript, 2010.

D. Bouyssou, T. Marchant, M. Pirlot, A. Tsoukiàs, and P. Vincke. *Evaluation and Decision Models: Stepping Stones for the Analyst*. Springer-Verlag, 2006.

F. Brandt. Minimal stable sets in tournaments. Technical report, http://arxiv.org/abs/0803.2138, 2009. Presented at the 9th International Meeting of the Society of Social Choice and Welfare.

F. Brandt and F. Fischer. Computing the minimal covering set. *Mathematical Social Sciences*, 56(2):254–268, 2008.

F. Brandt and P. Harrenstein. Characterization of dominance relations in finite coalitional games. *Theory and Decision*, 2011. Forthcoming.

F. Brandt, F. Fischer, P. Harrenstein, and M. Mair. A computational analysis of the tournament equilibrium set. *Social Choice and Welfare*, 34(4):597–609, 2010.

C. Capelle, M. Habib, and F. de Montgolfier. Graph decompositions and factorizing permutations. *Discrete Mathematics and Theoretical Computer Science*, 5:55–70, 2002.

V. Conitzer. Computing Slater rankings using similarities among candidates. In *Proceedings of the 21st National Conference on Artificial Intelligence (AAAI)*, pages 613–619. AAAI Press, 2006.

R. G. Downey and M. R. Fellows. *Parameterized Complexity*. Springer-Verlag, 1999.

J. Duggan and M. Le Breton. Dutta's minimal covering set and Shapley's saddles. *Journal of Economic Theory*, 70:257–265, 1996.

P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77:321–357, 1995.

P. E. Dunne. Computational properties of argumentation systems satisfying graph-theoretic constraints. *Artificial Intelligence*, 171(10-15):701–729, 2007.

B. Dutta. Covering sets and a new Condorcet choice correspondence. *Journal of Economic Theory*, 44:63–80, 1988.

S. L. Feld and B. Grofman. Who's afraid of the big bad cycle? Evidence from 36 elections. *Journal of Theoretical Politics*, 4:231–237, 1992.

P. C. Fishburn. Condorcet social choice functions. *SIAM Journal on Applied Mathematics*, 33(3):469–489, 1977.

D. C. Fisher and J. Ryan. Tournament games and positive tournaments. *Journal of Graph Theory*, 19(2):217–236, 1995.

G. Laffond, J.-F. Laslier, and M. Le Breton. The bipartisan set of a tournament game. *Games and Economic Behavior*, 5:182–201, 1993.

G. Laffond, J. Lainé, and J.-F. Laslier. Composition-consistent tournament solutions and social choice functions. *Social Choice and Welfare*, 13:75–93, 1996.

J.-F. Laslier. *Tournament Solutions and Majority Voting*. Springer-Verlag, 1997.

R. M. McConnell and F. de Montgolfier. Linear-time modular decomposition of directed graphs. *Discrete Applied Mathematics*, 145(2):198–209, 2005.

N. R. Miller. A new solution set for tournaments and majority voting: Further graph-theoretical approaches to the theory of voting. *American Journal of Political Science*, 24 (1):68–96, 1980.

H. Moulin. Choosing from a tournament. *Social Choice and Welfare*, 3:271–291, 1986.

R. Niedermeier. *Invitation to Fixed-Parameter Algorithms*. Oxford University Press, 2006.

T. Schwartz. Cyclic tournaments and cooperative majority voting: A solution. *Social Choice and Welfare*, 7:19–29, 1990.

T. N. Tideman. Independence of clones as a criterion for voting rules. *Social Choice and Welfare*, 4(3):185–206, 1987.

G. J. Woeginger. Banks winners in tournaments are difficult to recognize. *Social Choice and Welfare*, 20:523–528, 2003.

H. P. Young. Condorcet's theory of voting. *The American Political Science Review*, 82(4): 1231–1244, 1988.

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