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Lecture in Winter Semester 2022/2023

Test Exam - Algorithmic Game Theory

January 17th 2023

The tasks in the final exam can be similar to the tasks in the following test exam but they do not have to! The scope of the test exam does not have to correspond to the scope of the final exam! The test exam is based on the post-exam for Algorithmic Game Theory in winter semester 2015/2016.

Name:

Major, Semester:

Matrikelnummer:

Number of sheets handed in, including task sheets:

Allowed tools:

- notes from the lecture, books, exercise sheets,
- memory.

Not allowed tools:

- mobile phones and other communication devices,
- other students,
- calculators.

Be sure that your calculations and all steps of your solutions are complete and clear!

Task	1	2	3	4	5	6	Total
all points	20	25	15	15	15	10 + 10 (Bonus)	100 + 10
gained points							

Task 1 (20 points) *Multiple Choice*

Mark either „Yes“ or „No“ for each question below.

Evaluation: For $\#R$ being the number of the correct answers chosen by you and $\#K$ being the number of all answers marked by you (i. e. only these where *either „Yes“ or „No“* are marked – The answers with neither „Yes“ nor „No“ or both „Yes“ and „No“ marked are not counted in $\#K$), the total number of points for the exam task is calculated with the following formula:

$$\#R + \left\lfloor \frac{5 \cdot \#R}{\#K} \right\rfloor \text{ points if } \#K > 0, \text{ and } 0 \text{ points if } \#K = 0.$$

(a) Which of the following statements are true?

- ☐ Yes ☐ No A Nash equilibrium in pure strategies is always a Pareto-optimal strategy profile in a noncooperative game in normal form.
- ☐ Yes ☐ No There exists a strict (and therefore unique) Nash equilibrium in pure strategies in the „Battle of the Sexes“ game.
- ☐ Yes ☐ No In the „Battle of the Sexes“ game, all Nash equilibria in pure strategies are Pareto optima and vice versa.

(b) Which of the following statements are true?

- ☐ Yes ☐ No There exists at least one Nash equilibrium in mixed strategies in a noncooperative game in normal form.
- ☐ Yes ☐ No In the Prisoners' Dilemma, there exist more Nash equilibria in mixed strategies than there exist Nash equilibria in pure strategies.
- ☐ Yes ☐ No Each 3-simplex has six 1-faces and four 2-faces.

(c) Which of the following statements are provable true?

- ☐ Yes ☐ No It is possible to decide in deterministic polynomial time if player 1 has no winning strategy in the GEOGRAPHY game.
- ☐ Yes ☐ No You can always force a draw in the tic-tac-toe game.
- ☐ Yes ☐ No The 3-*player majority game* game $G = (P, v)$ (defined by $P = \{1, 2, 3\}$ and $v(C) = 1$ if $\|C\| \geq 2$, and $v(C) = 0$ otherwise) is superadditive.

(d) Which of the following statements are true?

- ☐ Yes ☐ No Convex cooperative games always have a non-empty core.
- ☐ Yes ☐ No Each subgame of a convex cooperative game has a non-empty core.
- ☐ Yes ☐ No Non-convex cooperative games always have an empty core.

(e) Which of the following statements are true?

- ☐ Yes ☐ No A simple superadditive game has a non-empty core if and only if it has no veto player.
- ☐ Yes ☐ No It is possible to decide in deterministic polynomial time if a given player in a given weighted voting game (in which the grand coalition wins) is a veto player.
- ☐ Yes ☐ No If a player has a Shapley value of 0 in a monotonic cooperative game, the player has to be a dummy player.

Task 2 (25 points) *Solution concepts in noncooperative games.*

Consider the noncooperative game in normal form with the set of players $P = \{1, 2\}$, the set of strategy profiles $\mathscr{S} = S_1 \times S_2$, where $S_i = \{a, b, c, d, e, f\}$ for $i \in \{1, 2\}$, and the following gain functions:

		Player 2					
		a	b	c	d	e	f
Player 1	a	$(1, 4)$	$(2, 3)$	$(2, 4)$	$(3, -1)$	$(2, -1)$	$(-2, 4)$
	b	$(1, 4)$	$(3, 2)$	$(-1, 3)$	$(1, -3)$	$(2, 4)$	$(4, 3)$
	c	$(-4, 1)$	$(3, 2)$	$(1, 2)$	$(-1, 4)$	$(-3, 2)$	$(-2, 1)$
	d	$(3, 4)$	$(-3, 2)$	$(-2, 1)$	$(1, -3)$	$(1, 1)$	$(3, 4)$
	e	$(4, 4)$	$(1, 3)$	$(1, 3)$	$(3, 3)$	$(2, -2)$	$(-1, 1)$
	f	$(1, 3)$	$(2, 2)$	$(2, 3)$	$(1, -3)$	$(1, -1)$	$(3, -3)$

(a) Determine all Nash equilibria in pure strategies and all strict Nash equilibria in pure strategies. Justify in each case why it is a Nash equilibrium or a strict Nash equilibrium, respectively.

(b) Consider the restriction of this game to the strategy sets $S_1' = \{a, b\}$ and $S_2' = \{a, b\}$ and determine all Pareto-optimal strategy profiles and all dominant strategies in the restricted game. (To justify the Pareto-optimality of a strategy profile, a graphical illustration can be used if it is explained in detail).

(c) Consider the restriction of this game to the strategy sets $S_1'' = \{e, f\}$ and $S_2'' = \{e, f\}$ and determine all Nash equilibria in mixed strategies. Argue briefly why there are no Nash equilibria other than the ones given.

(d) Consider the restriction of this game to the strategy sets $S_1'' = \{e, f\}$ and $S_2'' = \{e, f\}$ and determine the minmax strategy of player 2 against player 1 and the minmax value for player 1.

(Please give all intermediate steps of the calculations in all subtasks and justify your answers!)

Task 3 (15 points) *Properties in non-cooperative games*

Given a noncooperative game in normal form with two players and two strategies per player, in which all strategy profiles are Pareto-optimal. Show that if there are two Nash equilibria in pure strategies in such a game, both players have at least one dominant strategy.

(Please argue formally about the gain function and make sure your argument is complete and understandable!)

Task 4 (15 points) *Coalition structures and superadditivity*

Consider the cooperative game with transferable utility $G = (P, v)$ with $P = \{1, 2, 3\}$ and v given in the following table:

C	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(C)$	0	0	0	1	0	1	1	0

(a) Determine all possible outcomes (i. e. solutions in form (coalition structure, payoff vector)) of the game which maximaze the social welfare. Limit yourself to payoff vectors which consist of integers and are efficient and individually rational.

(b) Prove that the game is not superadditive.

(c) Determine the superadditive cover of the game.

(Please give all steps of your calculations in all tasks and justify your answers!)

Task 5 (15 points) *Solution concepts and power indices in cooperative games*

Consider the cooperative game with transferable utility $G = (P, v)$ with $P = \{1, 2, 3\}$ and

$$v(C) = \begin{cases} 1 & \text{if } 3 \in C \\ 0 & \text{otherwise} \end{cases}$$

for all $C \subseteq P$. All statements presented in the lecture can be used in this task.

- **(a)** Determine the Shapley values of the three players.
- **(b)** Prove that the game is convex.
- **(c)** Is the core of the game empty? Justify your answer. If the core is not empty, give an element from it and explain why this element belongs to the core. If the core is empty, explain in particular why $(0, 0, 1)$ does not belong to it.

(Please make sure that your argumentation is complete and clear, and present all steps of your solution!)

Task 6 (10 Punkte + 10 Bonuspunkte) *Properties of simple cooperative games*

Let $G = (P, v)$ be a simple game and let $\mathscr{W} = \{C \subseteq P \mid v(C) = 1\}$ be the set of all winning coalitions. G is called

- proper if $C \in \mathscr{W} \implies P \setminus C \notin \mathscr{W}$; and
- weak if $\bigcap_{C \in \mathscr{W}} C \neq \emptyset$.

Solve one of the following tasks your choice. If you solve both of the tasks, you will be able to get bonus points.

(a) Give a value for the quota q in a general weighted voting game $(w_1, \dots, w_n; q)$ with $n \geq 2$ which makes the game unproper. Explicitly explain why the given game is not proper. Prove that so defined weighted voting game is not convex.

(b) Let G be a superadditive, simple game. Prove that

$$G \text{ is weak} \Rightarrow \mathrm{CoS}(G) = 0.$$

(Please make sure that your argumentation is complete and clear. It will also be possible to get some points for not complete argumentation if it clear is!)

Please submit the task sheets with the headlines filled out.
HAVE FUN – VIEL SPASS – GOOD LUCK – VIEL GLÜCK – MAY YOU DO WELL – VIEL ERFOLG