

# Algorithmic Game Theory

Algorithmische Spieltheorie

Pingo

Wintersemester 2022/2023

Dozent: Prof. Dr. J. Rothe



## Website

<https://pingo.coactum.de/>

Access Number:  
885317



© Titanic Verlag

## Question 1

Consider the weighted voting game  $G = (2, 2, 2; 4)$ . In terms of the normalized Banzhaf index, is splitting into two players of equal weight for, say, the third player ...

- A ... beneficial?
- B ... neutral?
- C ... disadvantageous?

## Question 2

Consider the weighted voting game  $G = (2, 2, 2; 5)$ . In terms of the normalized Banzhaf index, is splitting into two players of equal weight for, say, the third player ...

- A ... beneficial?
- B ... neutral?
- C ... disadvantageous?

## Question 3

Consider the weighted voting game  $G = (2, 2, 2; 6)$ . In terms of the normalized Banzhaf index, is splitting into two players of equal weight for, say, the third player ...

- A ... beneficial?
- B ... neutral?
- C ... disadvantageous?

## Question 4

In all examples presented so far, weight-splitting had the same effect on the Shapley–Shubik index and the normalized Banzhaf index of the manipulator.

**Is this is always the case?**

A Yes

B No

C  (ö)

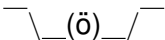
## Answer 4

In all examples presented so far, weight-splitting had the same effect on the Shapley–Shubik index and the normalized Banzhaf index of the manipulator.

**Is this is always the case?**

A Yes

**B No**

C 

## Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011)

Consider the WVG  $G = (2, 1, 1, 1, 1; 5)$ .



## Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011)

Consider the WVG  $G = (2, 1, 1, 1, 1; 5)$ .

In this game, the first player is pivotal for a permutation if he appears in the last or second-to-last position, but not in earlier positions.

## Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011)

Consider the WVG  $G = (2, 1, 1, 1, 1; 5)$ .

In this game, the first player is pivotal for a permutation if he appears in the last or second-to-last position, but not in earlier positions.

Thus, his Shapley–Shubik index is  $\frac{2}{5}$ .

## Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011)

Consider the WVG  $G = (2, 1, 1, 1, 1; 5)$ .

In this game, the first player is pivotal for a permutation if he appears in the last or second-to-last position, but not in earlier positions.

Thus, his Shapley–Shubik index is  $\frac{2}{5}$ .

Further, this player is pivotal for any coalition that contains three or four players of weight 1, i.e., for 5 coalitions.

## Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011)

Consider the WVG  $G = (2, 1, 1, 1, 1; 5)$ .

In this game, the first player is pivotal for a permutation if he appears in the last or second-to-last position, but not in earlier positions.

Thus, his Shapley–Shubik index is  $\frac{2}{5}$ .

Further, this player is pivotal for any coalition that contains three or four players of weight 1, i.e., for 5 coalitions.

On the other hand, any player of weight 1 is pivotal for any coalition that contains the player of weight 2 as well as any two other players of weight 1, i.e., for 3 coalitions.

## Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011; continued)

Thus the normalized Banzhaf index of the first player is given by

$$\frac{5}{5 + 4 \cdot 3} = \frac{5}{17}.$$

## Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011; continued)

Thus the normalized Banzhaf index of the first player is given by

$$\frac{5}{5 + 4 \cdot 3} = \frac{5}{17}.$$

It remains to observe that, after splitting the first player into two new players, we have for  $G' = (1, 1, 1, 1, 1, 1; 5)$ :

$$\begin{aligned} \frac{2}{5} &> \frac{1}{3} = \text{SSI}(G', 1) + \text{SSI}(G', 2) \quad \text{but} \\ \frac{5}{17} &< \frac{1}{3} = \overline{\text{BI}}(G', 1) + \overline{\text{BI}}(G', 2). \end{aligned}$$

## Question 5

Consider the weighted voting game  $G = (2, 2, 1, 1; 4)$ . In terms of the probabilistic Banzhaf index, is merging the last two players into one third player (yielding  $G' = (2, 2, 2; 4)$ ) ...

- A ... beneficial?
- B ... neutral?
- C ... disadvantageous?

## Question 6

Consider the weighted voting game  $G = (2, 2, 1, 1; 5)$ . In terms of the probabilistic Banzhaf index, is merging the last two players into one third player (yielding  $G' = (2, 2, 2; 5)$ ) ...

- A ... beneficial?
- B ... neutral?
- C ... disadvantageous?



## Question 7

Consider the weighted voting game  $G = (2, 2, 1, 1; 6)$ . In terms of the probabilistic Banzhaf index, is merging the last two players into one third player (yielding  $G' = (2, 2, 2; 6)$ ) ...

- A ... beneficial?
- B ... neutral?
- C ... disadvantageous?

## Question 8

Is merging two players always neutral in terms of the probabilistic Banzhaf index?

A Yes

B No

C  (ö)