Algorithmic Game Theory

Algorithmische Spieltheorie

Pingo

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Website

https://pingo.coactum.de/

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Question 1

Consider the weighted voting game $G = (2, 2, 2; 4)$. In terms of the normalized Banzhaf index, is splitting into two players of equal weight for, say, the third player ...

A ... beneficial?

B ... neutral?

C ... disadvantageous?

Question 2

Consider the weighted voting game $G = (2, 2, 2; 5)$. In terms of the normalized Banzhaf index, is splitting into two players of equal weight for, say, the third player . . .

A . . . beneficial?

B . . . neutral?

C . . . disadvantageous?

Question 3

Consider the weighted voting game $G = (2, 2, 2; 6)$. In terms of the normalized Banzhaf index, is splitting into two players of equal weight for, say, the third player ...

A ... beneficial?

B ... neutral?

C ... disadvantageous?

Question 4

In all examples presented so far, weight-splitting had the same effect on the Shapley–Shubik index and the normalized Banzhaf index of the manipulator.

Is this is always the case?

A Yes

B No

C ¯_(ö)_/¯

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Is this is always the case?

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B No

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Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011)

Consider the WVG $G = (2, 1, 1, 1, 1; 5)$.

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Further, this player is pivotal for any coalition that contains three or four players of weight 1, i.e., for 5 coalitions.

On the other hand, any player of weight 1 is pivotal for any coalition that contains the player of weight 2 as well as any two other players of weight 1, i.e., for 3 coalitions.

Explanation of Answer 4

Example (Aziz, Bachrach, Elkind, Paterson, 2011; continued)

Thus the normalized Banzhaf index of the first player is given by

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Thus the normalized Banzhaf index of the first player is given by

$$\frac{5}{5+4\cdot 3} = \frac{5}{17}.$$

It remains to observe that, after splitting the first player into two new players, we have for $G' = (1, 1, 1, 1, 1, 1; 5)$:

$$\begin{aligned} \frac{2}{5} &> \frac{1}{3} = \mathrm{SSI}(G', 1) + \mathrm{SSI}(G', 2) \quad \text{but} \\ \frac{5}{17} &< \frac{1}{3} = \overline{\mathrm{BI}}(G', 1) + \overline{\mathrm{BI}}(G', 2). \end{aligned}$$

Question 5

Consider the weighted voting game $G = (2, 2, 1, 1; 4)$. In terms of the probabilistic Banzhaf index, is merging the last two players into one third player (yielding $G' = (2, 2, 2; 4)$) ...

A ... beneficial?

B ... neutral?

C ... disadvantageous?

Question 6

Consider the weighted voting game $G = (2, 2, 1, 1; 5)$. In terms of the probabilistic Banzhaf index, is merging the last two players into one third player (yielding $G' = (2, 2, 2; 5)$) ...

A ... beneficial?

B ... neutral?

C ... disadvantageous?

Question 7

Consider the weighted voting game $G = (2, 2, 1, 1; 6)$. In terms of the probabilistic Banzhaf index, is merging the last two players into one third player (yielding $G' = (2, 2, 2; 6)$) ...

A ... beneficial?

B ... neutral?

C ... disadvantageous?

Question 8

Is merging two players always neutral in terms of the probabilistic Banzhaf index?

A Yes

B No

C ¯_(ö)_/¯