# Manipulation: Strategic Voting

## Example

Consider the Borda election with candidates $a$, $b$, and $c$ and the following votes:

| | Sincere Votes | | | | Strategic Votes | | |
|---|---|---|---|---|---|---|---|
| points : | 2 | 1 | 0 | | 2 | 1 | 0 |
| 5 votes : | $a$ | $b$ | $c$ | | $a$ | $b$ | $c$ |
| 5 votes : | $b$ | $a$ | $c$ | $\Rightarrow$ | $b$ | $c$ | $a$ |
| 1 vote : | $c$ | $a$ | $b$ | | $c$ | $a$ | $b$ |
| | Borda winner $a$ | | | | Borda winner $b$ | | |

# Variants of the Manipulation Problem

## Definition (Constructive Coalitional Manipulation)

Let $\mathcal{E}$ be some voting system.

**Name:** $\mathcal{E}$-CONSTRUCTIVE COALITIONAL MANIPULATION ($\mathcal{E}$-CCM).

**Given:**
- A set $C$ of candidates,
- a list $V$ of nonmanipulative voters over $C$,
- a list $S$ of manipulative voters (whose votes over $C$ are still unspecified) with $V \cap S = \emptyset$, and
- a distinguished candidate $c \in C$.

**Question:** Is there a way to set the preferences of the voters in $S$ such that, under election system $\mathcal{E}$, $c$ is a winner of election $(C, V \cup S)$?

# Variants of the Manipulation Problem

Remark: Variants:

- $\mathcal{E}$-DESTRUCTIVE COALITIONAL MANIPULATION ($\mathcal{E}$-DCM) is the same with "$c$ is not a winner of $(C, V \cup S)$."
- If $\|S\| = 1$, we obtain the single-manipulator problems:
  - $\mathcal{E}$-CONSTRUCTIVE MANIPULATION ($\mathcal{E}$-CM) and
  - $\mathcal{E}$-DESTRUCTIVE MANIPULATION ($\mathcal{E}$-DM).
- Voters can also be weighted (see next slide).
- These problems can also be defined in the "unique-winner" model.

# Variants of the Manipulation Problem

**Definition (Constructive Coalitional Weighted Manipulation)**

Let $\mathcal{E}$ be some voting system.

**Name:** $\mathcal{E}$-CONSTRUCTIVE (DESTRUCTIVE) COALITIONAL WEIGHTED MANIPULATION ($\mathcal{E}$-CCWM / $\mathcal{E}$-DCWM).

**Given:**
- A set $C$ of candidates,
- a list $V$ of nonmanipulative voters over $C$ each having a nonnegative integer weight,
- a list of the weights of the manipulators in $S$ (whose votes over $C$ are still unspecified) with $V \cap S = \emptyset$, and
- a distinguished candidate $c \in C$.

**Question:** Can the preferences of the voters in $S$ be set such that $c$ is a $\mathcal{E}$-winner (is not an $\mathcal{E}$-winner) of $(C, V \cup S)$?

# Some Basic Complexity Classes

## Definition

1. $\mathrm{FP}$ denotes the *class of polynomial-time computable total functions* mapping from $\Sigma^*$ to $\Sigma^*$.
2. $\mathrm{P}$ denotes the *class of problems that can be decided in polynomial time* (i.e., via a deterministic polynomial-time Turing machine).
3. $\mathrm{NP}$ denotes the *class of problems that can be accepted in polynomial time* (i.e., via a nondeterministic polynomial-time Turing machine).

# Some Basic Complexity Classes

Remark:

- Intuitively, $\mathrm{FP}$ and $\mathrm{P}$, respectively, capture feasibility/efficiency of computing functions and solving decision problems.
- $A \in \mathrm{NP}$ if and only if there exist a set $B \in \mathrm{P}$ and a polynomial $p$ such that for each $x \in \Sigma^*$,

  $$x \in A \iff (\exists w)\,[|w| \leq p(|x|) \text{ and } (x, w) \in B].$$

  That is, $\mathrm{NP}$ is the class of problems whose YES instances can be easily checked.
- Central open question of TCS: $\mathrm{P} \stackrel{?}{=} \mathrm{NP}$
- Examples of problems in $\mathrm{NP}$: SAT, TRAVELING SALESPERSON PROBLEM, VERTEX COVER, CLIQUE, HAMILTON CIRCUIT, ...

# NP in Ancient Times

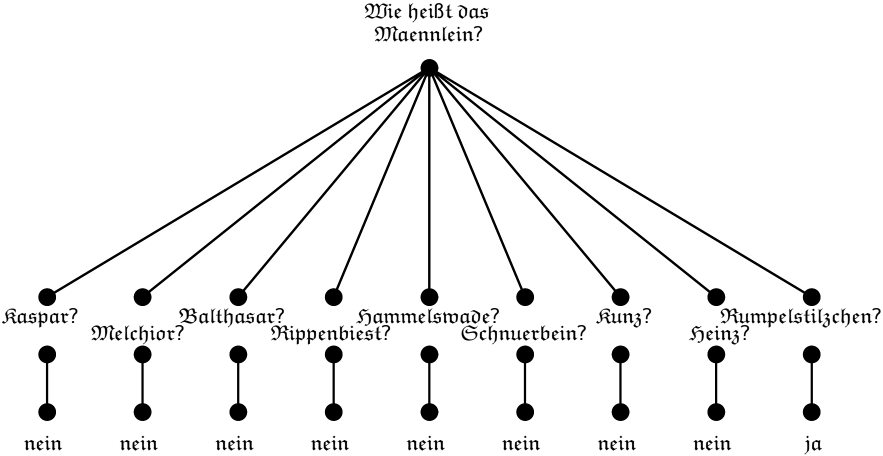


Figure: Nondeterministic Guessing and Deterministic Checking

# Pol-Time Many-One Reducibility and Completeness

## Definition

Let $\Sigma$ be an alphabet and $A, B \subseteq \Sigma^*$. Let $\mathcal{C}$ be any complexity class.

1. Define the *polynomial-time many-one reducibility*, denoted by $\leq_{\mathrm{m}}^{\mathrm{p}}$, as follows: $A \leq_{\mathrm{m}}^{\mathrm{p}} B$ if there is a function $f \in \mathrm{FP}$ such that
$$(\forall x \in \Sigma^*)\,[x \in A \iff f(x) \in B].$$
2. A set $B$ is *$\leq_{\mathrm{m}}^{\mathrm{p}}$-hard for $\mathcal{C}$* (or *$\mathcal{C}$-hard*) if $A \leq_{\mathrm{m}}^{\mathrm{p}} B$ for each $A \in \mathcal{C}$.
3. A set $B$ is *$\leq_{\mathrm{m}}^{\mathrm{p}}$-complete for $\mathcal{C}$* (or *$\mathcal{C}$-complete*) if
   1. $B$ is $\leq_{\mathrm{m}}^{\mathrm{p}}$-hard for $\mathcal{C}$ (lower bound) and
   2. $B \in \mathcal{C}$ (upper bound).
4. $\mathcal{C}$ is *closed under the $\leq_{\mathrm{m}}^{\mathrm{p}}$-reducibility* (*$\leq_{\mathrm{m}}^{\mathrm{p}}$-closed*, for short) if
$$(A \leq_{\mathrm{m}}^{\mathrm{p}} B \text{ and } B \in \mathcal{C}) \implies A \in \mathcal{C}.$$

# Properties of $\leq_{\mathrm{m}}^{\mathrm{p}}$

1. $A \leq_{\mathrm{m}}^{\mathrm{p}} B$ implies $\overline{A} \leq_{\mathrm{m}}^{\mathrm{p}} \overline{B}$, yet in general it is not true that $A \leq_{\mathrm{m}}^{\mathrm{p}} \overline{A}$.
2. $\leq_{\mathrm{m}}^{\mathrm{p}}$ is a reflexive and transitive, yet not antisymmetric relation.
3. $\mathrm{P}$ and $\mathrm{NP}$ are $\leq_{\mathrm{m}}^{\mathrm{p}}$-closed.
   That is, upper bounds are inherited downward with respect to $\leq_{\mathrm{m}}^{\mathrm{p}}$.
4. If $A \leq_{\mathrm{m}}^{\mathrm{p}} B$ and $A$ is $\leq_{\mathrm{m}}^{\mathrm{p}}$-hard for some complexity class $\mathcal{C}$, then $B$ is $\leq_{\mathrm{m}}^{\mathrm{p}}$-hard for $\mathcal{C}$.
   That is, lower bounds are inherited upward with respect to $\leq_{\mathrm{m}}^{\mathrm{p}}$.
5. Let $\mathcal{C}$ and $\mathcal{D}$ be any complexity classes. If $\mathcal{C}$ is $\leq_{\mathrm{m}}^{\mathrm{p}}$-closed and $B$ is $\leq_{\mathrm{m}}^{\mathrm{p}}$-complete for $\mathcal{D}$, then $\mathcal{D} \subseteq \mathcal{C} \iff B \in \mathcal{C}$.
   In particular, if $B$ is $\mathrm{NP}$-complete, then

$$\mathrm{P} = \mathrm{NP} \iff B \in \mathrm{P}.$$

# Plurality and Regular Cup are Easy to Manipulate

**Theorem (Conitzer, Sandholm, and Lang (2007))**

*Plurality-CCWM and Regular-Cup-CCWM are in* $\mathrm{P}$ *(for any number of candidates, in both the unique-winner and nonunique-winner model).*

Proof:

1. For plurality, the manipulators simply check if $c$ wins when they all rank $c$ first.
   - If so, they have found a successful strategy.
   - If not, no strategy can make $c$ win.
2. For the regular cup protocol (given the assignment of candidates to the leaves of the binary balanced tree), see blackboard. ❑

# Copeland with three Candidates is Easy to Manipulate

*Copeland voting*: For each $c, d \in C$, $c \neq d$,

- let $N(c, d)$ be the number of voters who prefer $c$ to $d$,
- let $C(c, d) = 1$ if $N(c, d) > N(d, c)$ and
- $C(c, d) = 1/2$ if $N(c, d) = N(d, c)$.
- The *Copeland score of c* is $CScore(c) = \sum_{d \neq c} C(c, d)$.
- Whoever has the maximum Copeland score wins.

**Theorem (Conitzer, Sandholm, and Lang (2007))**

*Copeland-CCWM for three candidates is in* $\mathrm{P}$
*(in both the unique-winner and nonunique-winner model).*

Proof: We show that: If Copeland with three candidates has a CCWM, then it has a CCWM where all manipulators vote identically.

And now...see blackboard.

# Maximin with three Candidates is Easy to Manipulate

*Maximin (a.k.a. Simpson) voting*: For each $c, d \in C$, $c \neq d$, let again $N(c,d)$ be the number of voters who prefer $c$ to $d$.

- The *maximin score of c* is

$$MScore(c) = \min_{d \neq c} N(c,d).$$

- Whoever has the maximum *MScore* wins.

**Theorem (Conitzer, Sandholm, and Lang (2007))**

*Maximin-CCWM for three candidates is in* $\mathrm{P}$
*(in both the unique-winner and nonunique-winner model).*

Proof: We show that: If Maximin with three candidates has a CCWM, then it has a CCWM where all manipulators vote identically.

And now. . . see blackboard.

# Upper bounds are inherited downward w.r.t. $\leq_m^p$

## Corollary

*All more restrictive variants of the manipulation problem are in* $\mathrm{P}$ *for:*

- *plurality (for any number of candidates),*
- *regular cup (for any number of candidates),*
- *Copeland (for at most three candidates), and*
- *maximin (for at most three candidates).*

# STV-CM is NP-complete

*Single Transferable Vote (STV)* for $m$ candidates proceeds in $m - 1$ rounds. In each round:

- A candidate with lowest plurality score is eliminated (using some tie-breaking rule if needed) and
- all votes for this candidate transfer to the next remaining candidate in this vote's order.

The last remaining candidate wins.

**Theorem (Bartholdi and Orlin (1991))**

STV-Constructive Manipulation *is* NP*-complete.*

# STV-CM is NP-complete: Reduction from X3C

Proof: Membership in $\mathrm{NP}$ is clear.

To prove $\mathrm{NP}$-hardness of STV-CONSTRUCTIVE MANIPULATION, we reduce from the following $\mathrm{NP}$-complete problem:

Name: EXACT COVER BY THREE-SETS (X3C).

Given:
- A set $B = \{b_1, b_2, \ldots, b_{3m}\}$, $m \geq 1$, and
- a collection $\mathcal{S} = \{S_1, S_2, \ldots, S_n\}$ of subsets $S_i \subseteq B$ with $\|S_i\| = 3$ for each $i$, $1 \leq i \leq n$.

Question: Is there a subcollection $\mathcal{S}' \subseteq \mathcal{S}$ such that each element of $B$ occurs in exactly one set in $\mathcal{S}'$?

In other words, does there exist an index set $I \subseteq \{1, 2, \ldots, n\}$ with $\|I\| = m$ such that $\bigcup_{i \in I} S_i = B$?

# STV-CM is NP-complete: The Candidates

Given an instance $(B, \mathcal{S})$ of X3C with

$$B = \{b_1, b_2, \ldots, b_{3m}\}$$
$$\mathcal{S} = \{S_1, S_2, \ldots, S_n\}$$

where $m \geq 1$, $S_i \subseteq B$ with $\|S_i\| = 3$ for each $i$, $1 \leq i \leq n$, construct an election $(C, V \cup \{s\})$ with manipulator $s$ and $5n + 3(m+1)$ candidates:

1. "possible winners": $c$ and $w$;
2. "first losers": $a_1, a_2, \ldots, a_n$ and $\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n$;
3. "$w$-bloc": $b_0, b_1, \ldots, b_{3m}$;
4. "second line": $d_1, d_2, \ldots, d_n$ and $\overline{d}_1, \overline{d}_2, \ldots, \overline{d}_n$;
5. "garbage collectors": $g_1, g_2, \ldots, g_n$.

# STV-CM is NP-complete: The Properties

Property 1: $a_1, a_2, \ldots, a_n$ and $\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n$ are among the first $3n$ candidates to be eliminated.

Property 2: Let $I = \{i \mid \overline{a}_i$ is eliminated prior to $a_i\}$. Then

$$c \text{ can be made win } (C, V \cup \{s\}) \iff I \text{ is a 3-cover.}$$

Property 3:

1. For any $I \subseteq \{1, 2, \ldots, n\}$, there is a preference for $s$ such that

$$\overline{a}_i \text{ is eliminated prior to } a_i \iff i \in I.$$

2. Such a preference for $s$ is constructed as follows:
   - If $i \in I$ then place $a_i$ in the $i$th position of $s$.
   - If $i \notin I$ then place $\overline{a}_i$ in the $i$th position of $s$.

# STV-CM is NP-complete: The Nonmanipulative Voters

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (1) | | $12n$ | votes: | $c$ | $\cdots$ | | | |
| (2) | | $12n-1$ | votes: | $w$ | $c$ | $\cdots$ | | |
| (3) | | $10n+2m$ | votes: | $b_0$ | $w$ | $c$ | $\cdots$ | |
| (4) | For each $i \in \{1,2,\dots,3m\}$, | $12n-2$ | votes: | $b_i$ | $w$ | $c$ | $\cdots$ | |
| (5) | For each $j \in \{1,2,\dots,n\}$, | $12n$ | votes: | $g_j$ | $w$ | $c$ | $\cdots$ | |
| (6) | For each $j \in \{1,2,\dots,n\}$, | $6n+4j-5$ | votes: | $d_j$ | $\overline{d}_j$ | $w$ | $c$ | $\cdots$ |
| | and if $S_j = \{b_x, b_y, b_z\}$ then | 2 | votes: | $d_j$ | $b_x$ | $w$ | $c$ | $\cdots$ |
| | | 2 | votes: | $d_j$ | $b_y$ | $w$ | $c$ | $\cdots$ |
| | | 2 | votes: | $d_j$ | $b_z$ | $w$ | $c$ | $\cdots$ |
| (7) | For each $j \in \{1,2,\dots,n\}$, | $6n+4j-1$ | votes: | $\overline{d}_j$ | $d_j$ | $w$ | $c$ | $\cdots$ |
| | | 2 | votes: | $\overline{d}_j$ | $b_0$ | $w$ | $c$ | $\cdots$ |
| (8) | For each $j \in \{1,2,\dots,n\}$, | $6n+4j-3$ | votes: | $a_j$ | $g_j$ | $w$ | $c$ | $\cdots$ |
| | | 1 | vote: | $a_j$ | $d_j$ | $g_j$ | $w$ | $c$ |
| | | 2 | votes: | $a_j$ | $\overline{a}_j$ | $g_j$ | $w$ | $c$ |
| (9) | For each $j \in \{1,2,\dots,n\}$, | $6n+4j-3$ | votes: | $\overline{a}_j$ | $g_j$ | $w$ | $c$ | $\cdots$ |
| | | 1 | vote: | $\overline{a}_j$ | $\overline{d}_j$ | $g_j$ | $w$ | $c$ |
| | | 2 | votes: | $\overline{a}_j$ | $a_j$ | $g_j$ | $w$ | $c$ |

# STV-CM is NP-complete: Elimination Sequence Encodes a 3-Cover

**Lemma (Bartholdi and Orlin (1991))**

1. *Exactly one of $d_j$ and $\overline{d}_j$ will be among the first $3n$ candidates to be eliminated.*

2. *Candidate $c$ will win if and only if*

$$J = \{j \mid d_j \text{ is among the first } 3n \text{ candidates to be eliminated}\}$$

*is the index set of an exact 3-cover for $\mathcal{S}$.*

**Proof:** See blackboard. ❑

# STV-CM is NP-complete: The Manipulor's Preference

**Lemma (Bartholdi and Orlin (1991))**

*Let $I \subseteq \{1, 2, \ldots, n\}$ and consider the strategic preference of manipulator $s$ in which the $i$th candidate is*

- $a_i$ *if* $i \in I$ *and*
- $\overline{a}_i$ *if* $i \notin I$.

*Then the order in which the first $3n$ candidates are eliminated is:*

1. *The $(3i - 2)$nd candidate to be eliminated is*
   - $\overline{a}_i$ *if* $i \in I$ *and*
   - $a_i$ *if* $i \notin I$.
2. *The $(3i - 1)$st candidate to be eliminated is*
   - $d_i$ *if* $i \in I$ *and*
   - $\overline{d}_i$ *if* $i \notin I$.
3. *The $3i$th candidate to be eliminated is*
   - $a_i$ *if* $i \in I$ *and*
   - $\overline{a}_i$ *if* $i \notin I$.

# {Scoring-Protocols without Plurality}-CCWM

**Theorem (Conitzer, Sandholm, and Lang (2007))**

{*Scoring-Protocols without Plurality*}-CONSTRUCTIVE COALITIONAL WEIGHTED MANIPULATION *for three candidates is* $\mathrm{NP}$*-complete.*

Remark:

1. For two candidates every scoring protocol is easy to manipulate.
2. Plurality is easy to manipulate for any number of candidates.
3. In particular, Veto-CCWM and Borda-CCWM for three candidates are $\mathrm{NP}$-complete.
4. The above theorem was independently proven by Hemaspaandra & Hemaspaandra (2007) and Procaccia & Rosenschein (2006).

# $\{$Scoring-Protocols without Plurality$\}$-CCWM: Reduction from PARTITION

Proof: Membership in $\mathrm{NP}$ is clear.

Let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ be a scoring protocol other than plurality.

To prove $\mathrm{NP}$-hardness of $\alpha$-CCWM, we reduce from the following $\mathrm{NP}$-complete problem:

**Name:** PARTITION.

**Given:** A nonempty sequence $(k_1, k_2, \ldots, k_n)$ of positive integers such that $\sum_{i=1}^{n} k_i$ is an even number.

**Question:** Does there exist a subset $A \subseteq \{1, 2, \ldots, n\}$ such that

$$\sum_{i \in A} k_i = \sum_{i \in \{1,2,\ldots,n\} - A} k_i \ ?$$

# $\{$Scoring-Protocols without Plurality$\}$-CCWM: Reduction from PARTITION

Given an instance $(k_1, k_2, \ldots, k_n)$ of PARTITION with $\sum_{i=1}^{n} k_i = 2K$ for some integer $K$, construct an election $(C, V \cup S)$ with $C = \{a, b, p\}$ and

| | Vote Weight | Preference | | |
|---|---|---|---|---|
| $V$ : | $(2\alpha_1 - \alpha_2)K - 1$ | $a$ | $b$ | $p$ |
| | $(2\alpha_1 - \alpha_2)K - 1$ | $b$ | $a$ | $p$ |

$S$ : For each $i \in \{1, 2, \ldots, n\}$, $(\alpha_1 + \alpha_2)k_i$

See blackbord for the proof of:

$(k_1, k_2, \ldots, k_n) \in$ PARTITION $\iff$ $p$ can be made win $(C, V \cup S)$. ❑

# Copeland-CCWM for four Candidates is Hard

**Theorem (Conitzer, Sandholm, and Lang (2007))**

*Copeland-*Constructive Coalitional Weighted Manipulation *for four candidates is* NP*-complete.*

Proof: Membership in NP is clear. To prove NP-hardness of Copeland-CCWM, we again reduce from Partition.

Given an instance $(k_1, k_2, \ldots, k_n)$ of Partition with $\sum_{i=1}^{n} k_i = 2K$ for some integer $K$, construct an election

$$(C, V \cup S)$$

with $C = \{a, b, c, p\}$ and the following votes in $V \cup S$.

# Copeland-CCWM for four Candidates is Hard

$V$ :

| Vote Weight | Preference |
|---|---|
| $2K + 2$ | $p \quad a \quad b \quad c$ |
| $2K + 2$ | $c \quad p \quad b \quad a$ |
| $K + 1$ | $a \quad b \quad c \quad p$ |
| $K + 1$ | $b \quad a \quad c \quad p$ |

$S$ : For each $i \in \{1, 2, \ldots, n\}$, $k_i$

See blackbord for the proof of:

$$(k_1, k_2, \ldots, k_n) \in \textsc{Partition} \iff p \text{ can be made win } (C, V \cup S). \quad \square$$

# Maximin-CCWM for four Candidates is Hard

**Theorem (Conitzer, Sandholm, and Lang (2007))**

*Maximin-*CONSTRUCTIVE COALITIONAL WEIGHTED MANIPULATION *for four candidates is* NP*-complete.*

Proof: Membership in NP is clear. To prove NP-hardness of Maximin-CCWM, we again reduce from PARTITION.

Given an instance $(k_1, k_2, \ldots, k_n)$ of PARTITION with $\sum_{i=1}^{n} k_i = 2K$ for some integer $K$, construct an election

$$(C, V \cup S)$$

with $C = \{a, b, c, p\}$ and the following votes in $V \cup S$.

# Maximin-CCWM for four Candidates is Hard

| | | Vote Weight | Preference |
|---|---|---|---|
| $V$ : | | $7K-1$ | $a \quad b \quad c \quad p$ |
| | | $7K-1$ | $b \quad c \quad a \quad p$ |
| | | $4K-1$ | $c \quad a \quad b \quad p$ |
| | | $5K$ | $p \quad c \quad a \quad b$ |
| $S$ : | For each $i \in \{1, 2, \dots, n\}$, | $2k_i$ | |

See blackbord for the proof of:

$$(k_1, k_2, \dots, k_n) \in \textsc{Partition} \iff p \text{ can be made win } (C, V \cup S). \quad ❑$$

# STV-CCWM for three Candidates is Hard

**Theorem (Conitzer, Sandholm, and Lang (2007))**

*STV-*Constructive Coalitional Weighted Manipulation *for three candidates is* NP*-complete.*

Proof: Membership in NP is clear. To prove NP-hardness of STV-CCWM, we again reduce from Partition.

Given an instance $(k_1, k_2, \ldots, k_n)$ of Partition with $\sum_{i=1}^{n} k_i = 2K$ for some integer $K$, construct an election

$$(C, V \cup S)$$

with $C = \{a, b, p\}$ and the following votes in $V \cup S$.

# STV-CCWM for three Candidates is Hard

| | | Vote Weight | Preference | | |
|---|---|---|---|---|---|
| $V$ : | | $6K-1$ | $b$ | $p$ | $a$ |
| | | $4K$ | $a$ | $b$ | $p$ |
| | | $4K$ | $p$ | $a$ | $b$ |
| $S$ : | For each $i \in \{1, 2, \ldots, n\}$, | $2k_i$ | | | |

See blackbord for the proof of:

$$(k_1, k_2, \ldots, k_n) \in \textsc{Partition} \iff p \text{ can be made win } (C, V \cup S). \quad \square$$

# Destructive Manipulation

**Definition (Destructive Coalitional Weighted Manipulation)**

Let $\mathcal{E}$ be some voting system.

**Name:** $\mathcal{E}$-DESTRUCTIVE COALITIONAL WEIGHTED MANIPULATION ($\mathcal{E}$-DCWM).

**Given:**
- A set $C$ of candidates,
- a list $V$ of nonmanipulative voters over $C$ each having a nonnegative integer weight,
- a list of the weights of the manipulators in $S$ (whose votes over $C$ are still unspecified) with $V \cap S = \emptyset$, and
- a distinguished candidate $c \in C$.

**Question:** Can the preferences of the voters in $S$ be set such that $c$ is not a $\mathcal{E}$-winner of $(C, V \cup S)$?

### Theorem (Conitzer, Sandholm, and Lang (2007))

*Let $\mathcal{E}$ be a voting system such that:*

- *Each candidate gets a numerical score based on the votes, and all candidates with the highest score win.*
- *The score function is monotonic: If changing a vote $v$ satisfies*

$$\begin{aligned} & \{b \mid v \text{ prefers } a \text{ to } b \text{ before the change}\} \\ \subseteq \quad & \{b \mid v \text{ prefers } a \text{ to } b \text{ after the change}\}, \end{aligned}$$

  *then a's score does not decrease.*
- *Winner determination in $\mathcal{E}$ can be done in polynomial time.*

*Then $\mathcal{E}$-DCWM is in $\mathrm{P}$.*

Proof: See blackbord. ❑

### Corollary (Conitzer, Sandholm, and Lang (2007))

*For any number of candidates,* DCWM *is in* $\mathrm{P}$ *for*

- *Borda,*
- *veto,*
- *Copeland, and*
- *maximin.*

Remark: Since destructive manipulation can be harder than constructive manipulation by at most a factor of $m - 1$ (where $m$ is the number of candidates), DCWM is in $\mathrm{P}$ for

- plurality and
- regular cup

for any number of candidates.

# STV-DCWM for three Candidates is Hard

**Theorem (Conitzer, Sandholm, and Lang (2007))**

*STV-DESTRUCTIVE COALITIONAL WEIGHTED MANIPULATION for three candidates is* NP*-complete.*

Proof: Membership in NP is clear. To prove NP-hardness of STV-DCWM, we again reduce from PARTITION.

Given an instance $(k_1, k_2, \ldots, k_n)$ of PARTITION with $\sum_{i=1}^{n} k_i = 2K$ for some integer $K$, construct an election

$$(C, V \cup S)$$

with $C = \{a, b, d\}$ and the following votes in $V \cup S$.

# STV-DCWM for three Candidates is Hard

| | | Vote Weight | Preference | | |
|---|---|---|---|---|---|
| $V$ : | | $6K$ | $a$ | $d$ | $b$ |
| | | $6K$ | $b$ | $d$ | $a$ |
| | | $8K - 1$ | $d$ | $a$ | $b$ |
| $S$ : | For each $i \in \{1, 2, \ldots, n\}$, | $2k_i$ | | | |

See blackbord for the proof of:

$(k_1, k_2, \ldots, k_n) \in \text{PARTITION} \iff$ $d$ can be made to not win $(C, V \cup S)$. ❑

# Overview: Results for CCWM

| # of Candidates | 2 | 3 | $\geq 4$ |
|---|---|---|---|
| Plurality | P | P | P |
| Regular Cup | P | P | P |
| Copeland | P | P | NP-complete |
| Maximin | P | P | NP-complete |
| Veto | P | NP-complete | NP-complete |
| Borda | P | NP-complete | NP-complete |
| STV | P | NP-complete | NP-complete |

Table: Results for Constructive Coalitional Weighted Manipulation

# Overview: Results for DCWM

| # of Candidates | 2 | $\geq 3$ |
|---|---|---|
| Plurality | P | P |
| Regular Cup | P | P |
| Copeland | P | P |
| Maximin | P | P |
| Veto | P | P |
| Borda | P | P |
| STV | P | NP-complete |

Table: Results for Destructive Coalitional Weighted Manipulation